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Use of Analogy in Viewing Physical Phenomena.*

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The student who seriously takes up the study of physical science finds confronting him an immense accumulation of recorded facts, extensive treatises dealing with the "principles of physics," long shelves filled with transactions of learned societies, and mathematical expositions that seem hopelessly beyond his comprehension. If he is not to be discouraged at the outset, he will select a relatively contracted area for his mental explorations. Furthermore, taste or circumstances will shape his purpose, more or less definitely, to study the science for its own development, for the instruction of others or for the practical end of its application to engineering. If with any of these objects in view, he is des-

*Read by title.

tined to worthily add his share to progress, he will ponder the full import of the word, οἰκονομία. This Greek word means management of a household. But the student of physics must translate it to signify the management of the mental household, or *brain economy*.

No mind, however powerful, can hold in memory more than a small fraction of its own, or the experiences of others, not even in a highly specialized branch of science. Now, indeed, even the principles of physical science have reached a list which is difficult to reproduce in thought. Therefore, it is imperative that along with the study of physical science should go the study of how the mental efforts may be economized. There should be constant endeavor to learn how to *potentially* understand much and throw the least burden upon memory.

By what methods, then, should the study of physics be pursued that with the best mental economy, the greatest acquirements may follow? In answer to this inquiry it is commonly asserted, that the attention should be directed to the laws and principles of the science, in the application of which the infinite particular cases, that would overburden the memory, may be cared for and comprehended. While admitting the full force of this reply it still appears to be incomplete and not fully satisfying. Moreover this reply is chiefly applicable to the so-called exact sciences in which phenomena have been embraced in a body of laws and principles. In the sciences of biology, geology, meteorology and their like, the principles and generalizations are few while phenomena that attract direct study are innumerable. In physical science, on the other hand, principles and laws are almost wholly embodied and expressed in systems of differential equations. If with the idea of mental economy in view, the attention is to be attracted chiefly to laws and principles, these would be found embodied best in equations, and the student of physics would become a student of mathematics—supplied perhaps with the tools of analysis but with a mind unfurnished with images of phenomena, and with his powers of creation and discovery undeveloped. Without hoping to give an adequate reply to the inquiry put, as to what methods should be used in the study of physics to secure the best mental economy, one phase of a partial answer will be discussed in the following pages. Our theme may be outlined in the statement: *That, in studying physical science, a fruitful brain*

economy will best result by establishing the habit of mentally associating phenomena, belonging in groups and classes, according to their natural physical connections and the analogies that exist among them.

The descriptive sciences, biology, geology, and their sub-classes, have been enriched by discoveries in physics and are constantly receiving assistance from her hands. In these sciences, in which mathematical deductions and the formulation of exact laws are seldom possible there goes, along with the unwearied accumulation of observations, an association of the similar, the like, the analogous. From the data associated together judgments are formed and the methods of nature are described. But are the descriptive sciences, with their borrowing, in debt to physics? If physics claims her own, may she not take in return from these sciences ideas and methods, as payment for the exact reasoning and laws that she gives? If physics is not to become barren in discovery, and end as a science of differential equations, we believe she needs to take and use the fertile, data-collecting and associative methods of the descriptive sciences. In these sciences, observation, comparison, and analogy are the constant fruitful aids to their development.

We shall now endeavor to show to any one entering upon the serious study of physical science, in a treatment that is illustrative rather than argumentative, the value of the methods suggested above.

MEANING AND NATURE OF ANALOGY.

Thought moves on in time, as a bee in space, in a generally tortuous course, passing from one mental image to another, from one abstraction to another. Memory joins the image of the moment with those of moments passed. A mental pleasure is felt, and the past and present are more securely linked in memory, whenever the mind discovers that one image or one concept has a likeness, in form, color, or character, with others that it has had before. And this pleasure and impress are fullest, when among many images or concepts, a single element is discovered, like honey found in flowers, that is common to many or all. Witness is ever given in speech and literature of the mind's tendency to seek among its concepts those elements that are like, similar, or analogous. Thus,

referring to the turning of water into wine, Milton won a prize in the University for writing the single line:

"The conscious water knew its Lord and blushed."

Acknowledgment was given to the merit of perceiving a beautiful and delicate analogy. Similitudes and analogies are employed by all the masters in literature. Shakespeare makes Polonius say,

"To thine own self be true,
And it must follow *as the night the day,*
Thou canst not then be false to any man."

Here the analogy of night following day, is used to illustrate and impress the certainty with which the result of not being false to any man, must follow, if one is true to himself.

The psychological value of these literary analogies, as Herbert Spencer has pointed out in his Essay on Style, is gain in economy of thought. They give, with the minimum attention to the symbols of thought, the maximum of mental impression. In popular jargon and much literature we find the words: likeness, similarity, analogy, loosely used to have the same significance. But when science replaces literature, all the clearness and precision that it is possible to give to the meaning of words must be observed.

When the word, *analogy,* is to be used carefully and in a scientific sense, it may be best defined as the exact translation of its Greek original, ἀνα', according to, and λο'γος, proportion; which signifies, equality of ratios, or an agreement between the relations of things to one another. By this it is to be understood that two or more objects are analogous; if, when certain elements, which define them, are mentally abstracted, these are found to be equal, or in numerical relations that are proportionate.

Thus, if from the objects A and B, elements a_1, a_2, which define A and elements b_1, b_2, which define B, are mentally abstracted, the objects, A and B, are strictly analogous, in a scientific sense, if $a_1 : b_1 :: a_2 : b_2$.

Analogies used more or less precisely in this scientific sense are frequently drawn, to give an illustration, between objects or physical events. Thus the ebb and flow of the tide is compared to the swinging of a pendulum. Here the mind abstracts from the object, tide, the elements, height of water level and time of day, and from the object, pendulum, the elements, elongation and time,

and the two are considered to be analogous, because height of water level has somewhat the same relation to time of day as has the elongation of the pendulum to its time of swing.

Literature employs with advantage loose analogies based upon equality of the relations of elements of an accidental character, and often quite special to the particular objects compared. But the analogies used in science, to have scientific value, must be chosen so that exactly definable proportions may be stated between relations of elements that may be themselves precisely defined. We are thus lead to inquire more fully what constitutes in a scientific sense a perfect analogy.

Let $P_1, P_2, P_3 \cdots P_n$ stand for a set of phenomena. If examined minutely, each of these phenomena will be found to consist of a limitless number of elements. In the evolutionary progress of physical science certain of these elements have been observed to be as repeatedly presented as the phenomena recur. Hence certain elements, belonging to any one of the phenomena, have been considered as being elements that are characteristic of the phenomenon, and therefore suitable to use in defining and in describing it. When a phenomenon is considered these characteristic elements are mentally abstracted from the rest. Then the elements and the relations found to exist between them are stated in precise language and the phenomenon thus becomes defined, and may be described to others. In the descriptive sciences, biology, geology, etc., the characteristic elements and the relations among them are generally defined verbally or graphically represented. As the relations cannot be given with numerical precision, these sciences cannot be classed as exact sciences and the analogies among their various phenomena can be drawn only loosely. In the physical sciences, however, the relations between the characteristic elements of a phenomenon can be expressed with numerical precision, as relations between constants, variables, and differential coefficients of variables; and the relations so expressed generally take the form of differential equations.

If the differential equations, expressing the relations that exist between the characteristic elements of each of the phenomena, $P_1, P_2, P_3 \cdots P_m$, are of the same form and number for each of the phenomena, then the phenomena form what may be termed a group of perfect analogies. By the same form, it is to be understood that they are alike in form, in respect to their variables,

to the derivatives of their variables and to the constants that figure in the different terms. The above statements will be illustrated and made clear by the following physical example:

Let Fig. 1 represent a right section of a cylinder of very great length and radius, R. Now this cylinder may become the seat of various phenomena, $P_1, P_2, P_3 \cdots P_n$, depending upon the assumptions that are made respecting it. By making proper assumptions some of the phenomena may be made to form a group of perfect analogies. Let assumptions be made that will yield a group of three analogous phenomena, P_1, P_2, and P_3. For this purpose conceive the cylinder to be filled with a conducting, incompressible fluid of density, ρ; mercury, for example, and confined in a very thin cylindrical wall.

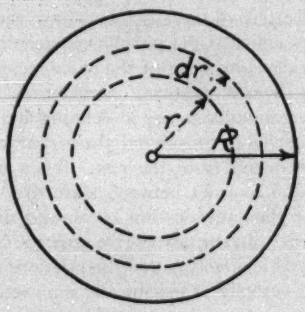

Fig. 1

Suppose the phenomenon, P_1, is the difference, g, in the hydrostatic pressures between the axis of the cylinder and its circumference; this difference in pressures being due solely to the gravitational attraction of the mass of the fluid. Suppose the phenomenon, P_2, is the difference, g_2, in the hydrostatic pressures between the axis of the cylinder and its circumference; this difference in pressures being due solely to the passage through the cylinder, parallel to its axis, of an electric current of current density I_1. Finally conceive the cylinder to revolve upon its axis with an angular velocity, ω, and let the phenomenon, P_3, be the difference, g_3, in the hydrostatic pressures between its circumfer-

ence and its axis; and that this pressure-difference is due solely to the centrifugal forces set up by the rotation of the cylinder.

Now it may be easily shown that for the phenomenon, P_1, we have,

$$dg_1 = -2\pi M^2 r\, dr, \tag{a}$$

or integrating from the axis to the circumference,

$$g_1 = -2\pi M^2 \int_0^R r\, dr = -\pi M^2 R^2 \tag{b}$$

In (a) and (b) M is the number of gravitational units of mass, per unit of volume, in the cylinder, and we are to take the negative sign to mean that the direction of the force acting to produce the pressure difference is in the opposite direction in which r is taken to increase, that is, the pressure is directed toward the axis.*

We have, similarly, for the phenomenon, P_2,

$$dg_2 = -2\pi I_1^2 r\, dr \tag{c}$$

or integrating from the axis to the circumference,

$$g_2 = -2\pi I_1^2 \int_0^R r\, dr = -\pi I_1^2 R^2 \tag{d}$$

Here the negative sign has the same significance as before.†

For the phenomenon, P_3, we may deduce the values of dg_3 and g_3 as follows:

The mass contained in an annular cylinder of unit length, of inside radius, r, and radial depth, dr, is,

$$dm = \rho \cdot 2\pi r\, dr$$

The centrifugal force exerted by the mass, dm, acting radially outward from the axis is,

$$dF = \omega^2 r \cdot 2\pi \rho r\, dr,$$

and the hydrostatic pressure, or pressure per unit of area at a distance, r, from the axis is,

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*See Forces in the Interior of an Electric Conductor, by E. F. Northrup, Phys. Rev., p. 490, June, 1907.

†See Ibid. p. 487.

$$\frac{d\,F}{2\,\pi\,r} = d\,g_3 = \omega^2\,\rho\,r\,dr \qquad (e)$$

If (e) is integrated from the axis to the circumference of the cylinder, we shall have the difference in pressures, g_3, between the circumference and the axis. Thus,

$$g_3 = \omega^2\,\rho \int_0^R r\,dr = \frac{\omega^2\,\rho\,R^2}{2} \qquad (f)$$

Here we are to understand by the positive sign that the direction of the pressure is that in which r increases, the pressure being greatest at the circumference.

Remembering that g_1, g_2, g_3, are *differences* in pressures, we can now write the following perfectly analogous relations, where A_1, A_2, A_3, and B_1, B_2, B_3 are constants.

$$dg_1 = -\,2\,\pi\,M^2\,r\,dr = A_1\,M^2\,r\,dr \qquad (g)$$
$$dg_2 = -\,2\,\pi\,I_1^2\,r\,dr = A_2\,I_1^2\,r\,dr \qquad (h)$$
$$dg_3 = \rho\,\omega^2\,r\,dr = A_3\,\omega^2\,r\,dr \qquad (i)$$

and

$$g_1 = -\,\pi\,R^2\,M^2 = B_1\,M^2 \qquad (j)$$
$$g_2 = -\,\pi\,R^2\,I_1^2 = B_2\,I_1^2 \qquad (k)$$
$$g_3 = \frac{\rho\,R^2}{2}\,\omega^2 = B_3\,\omega^2 \qquad (l)$$

In the equations (g)—(l), we have the differential and integral mathematical expressions for the three phenomena, P_1, P_2, P_3, which constitute a group of perfect analogies. The corresponding elements, dg_1, dg_2, dg_3, of the three phenomena are to be classed as effects that are due to certain causes. In the group chosen these effects are identities in respect to quality or nature, but of possible difference in magnitude.

M^2, I_1^2, ω^2 are the causes of the effects and form a group of corresponding elements. The constants, A_1, A_2, A_3 enter the equations in like manner as do the variables, r and dr.

In the integrated expressions, g_1, g_2, g_3, are corresponding elements, constituting the effects, M^2, I_1^2, ω^2, are corresponding elements constituting the causes, while B_1, B_2, B_3, are constants that enter the terms in the same manner.

By paraphrasing a statement of M. Petrovitch* we may generalize the above as follows:

Let P_1, P_2, P_3—P_n, constitute a group of n phenomena. Then form the table,

$$\begin{array}{cccc} \alpha_1 & \alpha_2 & \alpha_3 & \cdots \\ \beta_1 & \beta_2 & \beta_3 & \cdots \\ \gamma_1 & \gamma_2 & \gamma_3 & \cdots \\ \cdot & \cdot & \cdot & \cdots \end{array}$$

of all the elements used to define all the phenomena, including the constants, variables, and differential coefficients of variables. The jth line will now contain the elements of the phenomenon, Pj. Then the notion of a perfect analogy between all the phenomena of the group implies the possibility of passing from the equation of any one phenomenon of the group to any other phenomenon of the group, by changing without varying the order, all the elements of any line of the table into all the elements of any other line of the table. The elements in the same column constitute, then, corresponding elements.

As it will be helpful to the development of our subject to bring into contrast the mathematical and the physical methods of stating and using analogies, we shall now give in turn some illustrations of each method.

ANALOGIES REGARDED AS RELATIONS BETWEEN MATHEMATICAL EXPRESSIONS.

The conclusion will be drawn from the view given above, as to what constitutes a perfect analogy, that all phenomena are strictly analogous that can be expressed by the same equations, when for each phenomenon considered the proper significance is given to each of the symbols of the equations. We shall assume that different groups of analogous phenomena may be expressed by equations of different forms, and that these equations having different forms may be arranged and classified according to a system that is based upon the greater or less correspondence of their characteristic terms. A general scheme or system of differential equations might be thus formulated, into which may be fitted allied groups of analogous phenomena.

Such classifications or schemes of differential equations have

*La Mécanique des Phénomènes fondée sur les Analogies, p. 8.

been formulated, notably by M. Petrovitch in the suggestive work referred to above. He points out that the few fundamental notions needed for the formulation and understanding of such a scheme has value in giving a clear insight into physical relations, in suggestiveness for lines of research, and in offering a philosophical view of physical relations that is not without interest because of its generality. He says:*

"Such a scheme will be truly called a new branch of natural philosophy destined to progressively and indefinitely enlarge, the object of which will be the study of the general mathematical relations between causes and their effects disembarrassed from all particularities that specially attach to such or such particular kind of phenomenon."

A scheme of this character contemplates an extensive classification of analogous phenomena into groups, in which each group is expressed by one or more differential equations of particular forms, the equations giving the relations that maintain between what are judged to be the characteristic elements of the phenomena. To make this manner of viewing physical relations quite clear we shall give examples of a few prominent groups of analogous phenomena, as defined by the equations which express the relations that exist between their characteristic elements.

A presentation of the physical method of viewing analogies will next be given in the form of a few examples. We shall then see how the physical method may be used to bring into view important aspects belonging to several phenomena, and we shall be in a position then to examine into the relative advantages that attach to the mathematical and to the physical methods.

From a general mathematical point of view, all phenomena may be considered as being the relations which maintain between quantities used to express causes and effects, by which the phenomena are characterized.

The characteristic elements which are mentally abstracted from phenomena and designated as the *cause*, or *causes*, of the phenomena, may be classified as forces; mechanical, electrical, magnetic, gravitational, chemical, etc.; and temperature differences. The forces may be simple forces, as a mechanical pull, or generalized forces, in the Lagrangian sense, as the moment of a couple,

*Ibid, p. 22.

a surface tension, a temperature difference, a variation of potential, etc. They may appear directly in the equations, or as explicit functions of other quantities, as time, space, gravitating matter, electric currents, and charges, magnetized matter, heat, linear or angular velocity, etc.

The characteristic elements mentally abstracted from phenomena and usually called *effects*, are: the variation with time or space of magnitudes, designated cöordinates, as a length ,a surface, a volume, an angle, magnetic flux, current flux, heat flux, etc. When we speak of phenomenal effects the idea of change or variation is usually present to the mind. Thus in speaking of the Hall effect, we think of the deflection of an electric current within a conductor. In the Thomson effect there is the evolution or absorption of heat caused by the electric current in flowing from one point in the conductor to another at a different temperature. Likewise the Peltier effect is the *change* in temperature produced at the junction of two dissimilar metals through which current passes.

If the effect is a variation in a quantity which varies only as a space cöordinate is made to vary, the notion of time not being involved, though the effect produced by the cause or causes may assume an infinite variety of forms, it is always something accomplished by forces acting through space, and hence may be considered as some form of work. Thus the change in level of mercury in a barometer, when this is taken to different heights, is a phenomenon in which the effect is the variation of the mercury column; work being done on or by the mercury as the space cöordinate, elevation above sea level, is changed.

Now following, in part, a method of procedure suggested by Petrovitch, write the following equation:

$$\varrho \frac{d\,\alpha}{d\,\mu} - (F_1 + F_2 + F_3 + \text{---}) = 0 \qquad (1)$$

The mathematical form of this equation indicates that a certain magnitude, α, varies in respect to another magnitude μ and, that this variation of α in respect to μ, is proportional to the sum of the quantities, F_1, F_2, F_3- - - - etc. We could with greater generality express the variation in α as the sum of the variations of its three rectangular components, but for the sake of simplicity we shall

treat a as a function which varies only when a single coördinate varies.

We shall take in this equation F_1, F_2, F_3, etc., to represent the numerical magnitudes of any finite number of simultaneously acting causes which produce a single effect, which is the variation of an element, a, mentally abstracted from a phenomenon, when an arbitrary variation occurs in a chosen magnitude, μ. The quantity μ will be chosen to vary uniformly, so that $d\mu$ is always constant.

The element a, mentally abstracted from a phenomenon, may be either what we shall designate, a primary element or a derived element. By primary element we shall understand one that cannot be further decomposed in thought. It may then be treated as a quantity, representing an elementary concept, as space, a quantity of electricity, or a quantity of heat. It is then a single coördinate that varies only when μ varies. When a is a derived element it will be a quantity which is related by one or more equations of union to another element taken as the primary element. If there is but one equation of union a will have the character of a velocity, or a current of electricity. It will then be the variation of the primary element from which it is mentally derived, as a current of electricity is the variation of a quantity of electricity with time. Its first derivative in respect to μ will equal the second derivative in respect to μ of the primary element. Hence when a is a derived element there exists at least one invariable relation as,

$$a\,d\,\mu - d\,\varepsilon = 0 \qquad (2)$$

in which ϵ designates a primary element mentally abstracted from any phenomenon and stands for a coördinate or numerical value that can be measured in terms of a unit of like kind.

When a is a derived element, joined with ϵ by equation (2) we shall call Ω, in analogy with mechanics, a *coefficient of inertia*. When a is a primary element we shall call Ω a *coefficient of restraint*. The full significance of the designations given above will appear in what follows.

Equation (1) is instinctively felt to present a relation that holds generally in nature, inasmuch as it is constantly observed that the magnitudes of effects are proportional to the sum of the intensities of all causes which simultaneously act to produce them. In the form, however, in which the equation stands it is entirely general

and without definite physical significance. But according to the designations given to the different symbols and the choice made of the causes, F_1, F_2, F_3, etc., it may be made to assume different forms, each of which will stand for a group of analogous phenomena. Before giving illustrations of some of the various forms the equation may take and the groups of phenomena embodied in them it will be found advantageous to make a few simple transformations of this equation, upon which some comments may be profitably made. In making the transformations, use will be made of the invariable relation given in equation (2).

From equation (2), $$\alpha = \frac{d\varepsilon}{d\mu}$$

and, $$\frac{d\alpha}{d\mu} = \frac{d^2\varepsilon}{d\mu^2}$$

Transpose the second term of Eq. (1), multiply both members by $d\varepsilon$, and replace $\frac{d\alpha}{d\mu}$ by its value $\frac{d^2\varepsilon}{d\mu^2}$, then integrate the resulting equation when we shall have

$$\Omega\left(\frac{d\varepsilon}{d\mu}\right)^2 = 2\int (F_1 + F_2 + F_3 + \cdots)\, d\varepsilon + c_1 \qquad (3)$$

or,

$$\Omega\,\alpha^2 = 2\int (F_1 + F_2 + F_3 + \cdots)\, d\varepsilon + c \qquad (4)$$

where c is the arbitrary constant of integration.

Now, assume that $F_1 + F_2 + F_3 + \cdots$ can be expressed by a single quantity, F, and, further, that $F\, d\epsilon$ is the exact differential of an arbitrary function Π. Then $\Pi = \int F\, d\epsilon + c_1$, and we shall have,

$$\Omega\,\alpha^2 = 2\Pi + K \qquad (5)$$

where c_1 and K are arbitrary constants resulting from the integration. Equation (5) is the integral form of the "theorem of living forces" first given by Laplace.* The quantity, $-\Pi$, is now known under the name of the potential function.

Equations (1) and (5) embody the mathematical relations that maintain between the characteristic elements of a vast num-

*Livre I, Sec. 19, of the Traité de Mécanique Céleste.

ber of physical phenomena. But before they can become available for expressing physical relations they must be made to assume particular forms that are based upon particular assumptions. Any particular form of equations (1) or (5) will express a group of analogous physical phenomena, and by giving particular meanings to the symbols of any particular form of these equations the relations between the characteristic elements of a particular phenomenon may be expressed.

In order to assign particular forms to equation (1), we must first consider, briefly, the general character of the causes, F_1, F_2, F_3, etc., upon which the effects depend. These causes may be broadly divided into *active* causes and *passive* causes. By active causes we shall understand those which act to initiate or continue the effect, $\Omega \frac{d\ a}{d\ \mu}$, and when proceeding from sources external to the object of the phenomenon will be expressed by giving to F_1, F_2, F_3, etc., a positive sign. Such causes will be mechanical, gravitational, electric and magnetic forces, temperature, light and sound intensities, etc.

By passive causes is meant those which act in direct opposition to the active causes and hence tend to diminish the effect. These will be expressed by giving F_1, F_2, F_3, etc., a negative sign. Such passive causes may be external to, or contained in, the object of the phenomenon, or both. Examples of these are, molecular and mechanical friction, magnetic damping, electrical resistance, magnetic reluctance, electric and magnetic hysteresis, resistance to heat conduction, thickness of a medium which has opacity for the transmission of any kind of radiant energy; or in general terms, anything which tends to bring about the final cessation of an effect.

The active causes may be constant in their manifestation, or, they may vary, being functions of a quantity which is taken as the independent variable. This last may express such concepts as time, space, mass, electricity, magnetism, temperature, heat, etc.

The passive causes may likewise be constant, or functions of the element a, or its differential coefficients, or both.

In nature phenomena, as observed on the earth, never occur without the presence, to a greater or less extent, of passive causes; this statement being only another way of saying that perpetual motion is impossible, for all effects, initiated by active causes,

must finally cease through the action of passive causes. It follows that any mathematical expression which expresses the relations between what are considered to be the characteristic elements of phenomena and which contains no terms expressive of passive causes, can only be a more or less close approximation to the relations actually existing on the earth. Equation (5) is such an expression. It appears very generally in the treatment of mechanical problems and in discussions upon theoretical mechanics. We may therefore pause in our discussion to make the following remarks: If we differentiate Equation (5), we have,

$$\mathrm{d}\frac{\Omega\, a^2}{2} = \mathrm{d}\, \Pi = (\mathrm{F}_1 + \mathrm{F}_2 + \mathrm{F}_3 + \text{- - -})\, \mathrm{d}\, \epsilon \qquad (6)$$

This states that the variation of $\frac{\Omega\, a^2}{2}$, which is an expression for kinetic energy, exactly equals the variation of a function of the forces, Π, or of the potential function $-\Pi$, it being remembered that this function can only exist when the forces depend exclusively upon variables that determine the state of the system. In equations (5) and (6) only one such variable, ϵ, is considered.

Now it is evident from observations made upon any mechanical or electrical system, that when the kinetic energy of the system varies, though the greater part of the energy transformed may spend itself in doing work against the forces in a manner to alter the potential of the system, a part of the transformed energy is always used up in overcoming frictional resistances and so converted into heat, and thereby lost to the system. This latter portion of the transformed energy is not embodied in equation (6). In the motions of the heavenly bodies only can it be said strictly, that the increase or the diminution of the kinetic energy of the system is closely or exactly equal to the diminution or augmentation of the internal potential of the system. In other words, in all phenomena that are observed to take place upon the earth there is, in strictness, no such principle as "conservation of living forces."

Many examples might be deduced to illustrate and prove the above statements, but the following familiar case only can be mentioned. Suppose a pendulum bob is pushed to one side from the vertical and then left free to oscillate. We should have the bob return, if equation (5) held strictly, to the lowest position

with a velocity that would carry it again to a point as high as that from which it descended. In its upward swing it would do work against the force of gravity and increase the internal potential of the system, pendulum bob and earth. This increase in potential would be sufficient so that in again diminishing to a minimum, the acquired kinetic energy would keep the bob oscillating forever. This is observed not to be the case in this instance, or any like it, and hence the approximate character of equation (5) and all physical reasoning based upon it.

By the use of proper assumptions equation (1) may be made to clearly express many groups of physical phenomena, some of which we shall now illustrate:

In equation (1) assume that α is a primary element and that only one cause, F_1, which is constant, is present. Equation (1) will then take the form

$$\frac{d\,\alpha}{d\,\mu} = \frac{F_1}{\Omega}$$

As F_1, and Ω are taken as constant quantities, the first member is constant, and calling this γ, we have,

$$\gamma = \frac{F_1}{\Omega} \tag{7}$$

This expression, identical with Ohm's law, simply states an intuitive perception that an effect manifests itself directly as the intensity of the cause producing it and inversely as the restraint furnished to its occurrence. As α is here chosen as a primary element, the effect designated by γ will be the uniform change with time of an element incapable of decomposition in thought. Thus γ will be a current of matter, a current of electricity, a magnetic flux, or a flux of heat, in which cases we shall have respectively, F_1, a mechanical force or pressure, an electromotive force, a magnetomotive force, a difference of temperature, and Ω, a mechanical resistance, an electrical resistance, the reciprocal of heat conductivity.

If α is now assumed to be a derived element, a large group of simple phenomena, which are entirely analogous, will be approximately symbolized if we give equation (1) the form

$$\Omega \frac{d\,\alpha}{d\,\mu} = F_1 \tag{8}$$

In this case F_1, is a constant active cause and both the equations (2) and (5) hold.

By integrating equation (8), we get

$$\Omega\, a = F_1\, \mu + A_1 \tag{9}$$

and as $a = \dfrac{d\,\varepsilon}{d\,\mu}$, we obtain in integrating Eq. (9)

$$\Omega\, \varepsilon = \tfrac{1}{2}\, F_1\, \mu^2 + A_1\, \mu + A_2 \tag{10}$$

where A_1 and A_2 are constants of integration.

It will be noted that as $\dfrac{d\,a}{d\,\mu} = \dfrac{d^2\,\varepsilon}{d\,\mu^2}$, the effect partakes of the character of an acceleration. If the constant cause, F_1, is some species of force, as the tension of a rubber band, gravitational, electric, or magnetic force, then Ω is a coefficient which relates in some manner to the inertia of whatever the force acts upon. Now inertia is a property which belongs only to matter and ether (unless we extend the property to electrons) and Equations (5), (8), (9), and (10) can only be extended, with the assumptions made, to mechanical and ether phenomena.

If we assign the symbols the meanings,

Ω = Mass of particle
ϵ = Distance of vertical descent
a = Velocity of descent
μ = Time
F_1 = Weight of particle = $\Omega\, g$
II = $\Omega\, g\, \epsilon$, or work done by gravity;

then Eq. (8) becomes $\dfrac{d\,a}{d\,\mu} = g$ and Eq. (10) becomes

$$\varepsilon = \tfrac{1}{2}\, g\, \mu^2 + A_1\, \mu + A_2 \tag{11}$$

which is recognized as expressing the law of bodies falling in vacuum. From Equation (5) we have,

$$\frac{\Omega\, a^2}{2} = \omega\, g\, \varepsilon + K_1 \tag{12}$$

which states that the kinetic energy of the falling body is equal to the work done by gravity on the body in moving it a distance, ϵ, plus a constant which expresses the kinetic energy possessed by the body when ϵ is zero.

These same equations, (8) to (12), with the proper concepts attached to the symbols, will give the angular acceleration, and angle turned through of a body rotating about a fixed axis and subjected to a constant torque. Also the electric current produced by applying a constant E.M.F. to a coil of wire having self induction but no ohmic resistance.

The analogous phenomena of any group expressed by an equation can always be put in evidence by means of a table. A few of the phenomena dependent upon Equations (8) to (12) are given in table I, which will serve as a sample of how such tables may with great advantage be constructed:

TABLE I

PHENOMENON	Q	ε	a	μ	$\frac{da}{d\mu}$	F_1	Π
Linear motion of a particle.	Mass of particle.	Linear distance.	Linear velocity.	Time.	Linear acceleration.	Constant force acting.	$F_1 \varepsilon =$ work done by force.
Circular motion of a particle distant, r, about a center, O.	Moment of inertia about O.	Length of arc.	Angular velocity	Time.	Angular acceleration.	$Fr =$ Moment of force.	$F \theta r =$ work done by force.
Flow of electric current in a coil of wire that has no resistance.	Self-Induction of coil.	Quantity of electricity.	Instantaneous Value of Current.	Time.	Change in current with the time.	Constant E. M. F.	$E \int d\varepsilon =$ work in moving Elec. in circuit.

Attention should be directed again to the facts that in nature, at least upon the earth, bodies do not fall in a perfect vacuum nor rotate on a frictionless axis, and current never flows in a coil entirely devoid of ohmic resistance, and in general, that the work done by forces is never wholly expended in developing kinetic energy that remains with the system. Some of this work is expended always against passive forces as friction, resistance, hysteresis, etc. The conclusion is therefore inevitable that any mathematical expression, or system of mechanics, which states that the effects produced in phenomena are functions of active forces only, is approximate in its fundamental character.*

*In this connection one should read Chapter XII of L'Evolution de la Méchanique, par P. Duhem.

Consider next the group of analogous phenomena in which a single cause acts, which is directly proportional to the element, a. Then $F_1 = -Ka$ and Equation (1) assumes the form,

$$\Omega \frac{da}{d\mu} = -Ka \tag{13}$$

where K is a constant.

In integrating Equation (13) we obtain

$$a = Ae^{-\frac{K}{\omega}\mu} \tag{14}$$

where A is a constant and e the base of the natural logarithms. In Equation (13) the effect, of whatever nature, is to be taken as the variation in the characteristic element, a, chosen as a coördinate, when the magnitude μ is made to vary arbitrarily.

If a is a primary element, that is if $\frac{da}{d\mu}$ is of the nature of a velocity, then Ω is a coefficient which determines the constant restraint put upon the effect to take place. In this case the instantaneous value of the effect will be directly proportional to the instantaneous value of the passive cause and inversely proportional to the magnitude of Ω, namely, to the constant restraint imposed upon the primary element to change.

If a is a derived element, that is, if $\frac{da}{d\mu}$ is of the nature of an acceleration, then Ω is a constant coefficient, which determine the ratio of the cause to the effect, and may therefore be taken as a coefficient of inertia. In this case, as the existing phenomenon can only have been initiated by an *active* cause, and as Equation (13) contains no expression for an active cause, $-Ka$ must wholly exert itself in causing a phenomenon previously produced to diminish or die out.

The law of an effect, when the only cause present is a passive cause which is proportional to the element that varies, is always expressed by a logarithmic curve, a graph of which is given in Fig. 2.

Now no event occurs upon the earth in which there is not present to a greater or less extent a passive cause that tends to make disappear a phenomenon which has been previously initiated by an

active cause. The following will serve as an illustrative example: Conceive a copper disk rotating in its own plane and turning upon its own axis between the poles of an electromagnet. Let Ω be the moment of inertia of this disk. At time, $\mu = 0$, let the disk be rotating with an angular velocity, α. It can only have attained this angular velocity through the agency of an active cause which is supposed to have ceased to act when $\mu = 0$. At this time let a passive cause begin to slow down the rotation. This passive cause will result from the magnetic flux produced by the electromagnet, through the plane of the disk, and will equal $-K\alpha$.

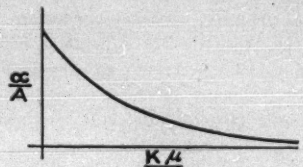


Fig. 2

If at time $\mu = 0$, the angular velocity is V, then by equation (14) the angular velocity of the disk at any time, μ, is

$$\alpha = V e^{-\frac{K\mu}{\omega}} \tag{15}$$

Here the constant, K, measures the effectiveness of the magnetic flux in producing a damping effect. At any time, μ, the angular velocity will be greater as the moment of inertia, Ω, is greater; that is the greater the coefficient of inertia, the less effective is the action of a passive cause. Now the damping effect may be very weak, resulting from a slight air friction only, but in time it is always manifested and always in one direction; namely, in opposition to the active cause which originated the motion.

Newton's law of heating or cooling of a body in still air is, in effect, that the temperature gained or lost in an element of time is proportional to the temperature difference between the body and the surrounding medium. Call α the temperature of the body above that of the medium at any time, μ. Then the variation of α as μ varies will be proportional to α itself. This relation is ex-

pressed in Equation (13). Now the effect as expressed by the law, is the variation in the temperature. The cause of this effect is the temperature gradient between the body and the medium. α will be a primary element as temperature cannot well be conceived as dependent upon a more fundamental quality. If Ω is taken as a constant quantity which is proportional to the specific heat of the body, the change in temperature with the time will be inversely as the magnitude of this quantity. We may call it therefore with propriety, a coefficient of restraint, inasmuch as it is a quantity which tends to hold back the variation in the primary element, α.

The equations (13) and (14) symbolize a group to which very many phenomena belong. Of these we may mention, merely the few following:

The variation of the barometer with the elevation;

The absorption with the time of electricity in the dielectric of a long insulated cable, after the capacity charging current has ceased, the conductor being joined to one pole of a battery, the other pole being grounded;

The variation with the distance from the generator, of the ratio of the entering current to the current at any point from the generator, on a very long telegraph line, grounded at the far end and carrying an alternating current;*

The variation with the time of the quantity of electricity which flows from a charged body earthed through a very high resistance;

A little reflection would suggest many other phenomena belonging to this group which might be mentioned.

It may be remarked that in general, any functions when multiplied by e^{-x} denotes an attenuation of the function as x increases and expresses thereby the inevitable trend of natural phenomena.

The passive cause, F_1, instead of being directly proportional to the element, α, which varies, might be proportional to any function of α. Thus the decrease in the angular velocity of the copper disk, above mentioned, due to air damping only, would be more

*See Eq. 24, p. 779, Elec. World and Eng., Nov. 15, 1902, art. by E. F. Northrup on "The Use and Advantages of the Alternating Current for Land Telegraphy."

nearly proportional to a^2. It would not, however, be of interest to examine here any of these functions.

An important group of phenomena occur in which the effect is produced by the simultaneous action of a constant active cause and a passive cause which is proportional to the characteristic element. In this case, if we first consider the characteristic element, a, as a derived element, Ω will be a coefficient of inertia and Equation (2) will hold. We shall then have, $F_1 =$ a constant active cause, or force, and $F_2 = -Ka$, so that Equation (1) assumes the form,

$$\omega \frac{d\,a}{d\,\mu} = F_1 - K\,a \qquad (16)$$

The well known integral of Equation (16) is

$$a = \frac{F_1}{K} + C\,e^{-\frac{K\,\mu}{\omega}} \qquad (17)$$

where C is the constant of integration.*

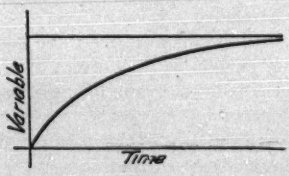


Fig. 3

A feature to be noted in this group of phenomena is that the characteristic element, a, tends to reach the constant value, $\frac{F_1}{K}$, as the independent variable, μ, increases, and approaches this value asymtotically. The graph, Fig. 3, shows these features:

A typical phenomenon of this group is the motion of a particle, of mass Ω, subjected to the accelerating action of a constant force, F_1, and moving in a medium which produces a retarding effect which is proportional to the instantaneous velocity of the particle. $\frac{d\,a}{d\mu}$ is the acceleration of the particle where μ is the time. The case

*Lamb's Infinitesimal Calculus, p. 475.

would also be exactly met by a long strip of copper falling in the direction of its length between the poles of a magnet.

The following mechanical phenomenon will serve as another illustration of this group. Let the armature of a rotary field induction motor be direct connected to a heavy fly wheel which together with the armature has a moment of inertia, Ω. Let μ be the time elapsed after the armature and fly wheel begin to rotate. Let the magnetic field rotate with the constant angular velocity V. Let α, which it is required to find, be the angular velocity of the fly wheel at any instant, μ. Then the torque on the armature will be proportional to the difference between the angular velocity of the field, and the armature, or the torque equals $K(V-\alpha)$. The inertia of the rotating parts will act as a force in opposition to the torque, and equilibrium must maintain between these forces, so we shall have,

$$-\omega \frac{d\alpha}{d\mu} + K(V - \alpha) = 0$$

or

$$\omega \frac{d\alpha}{d\mu} = KV - K\alpha \qquad (18)$$

Since K V is a constant, Equation (18) is of exactly the same form as Equation (16).

Integrated, Equation (18) gives

$$\alpha = Ce^{-\frac{K\mu}{\omega}} + V$$

If $\alpha = 0$ when $\mu = 0$, then $C = -V$ and

$$\alpha = V\left(1 - e^{-\frac{K\mu}{\omega}}\right) \qquad (19)$$

Thus the fly wheel attains its angular velocity, which is finally equal to that of the rotary field, according to the law shown in the curve, Fig. 3.

When a constant E.M.F. is suddenly applied to the terminals of a coil having inductance and resistance only, the electric current in the coil increases with the time by the same law as expressed in Equation (16). In this case Ω is the coefficient of magnetic inertia, or of self induction of the coil, F_1, the constant E.M.F. and K, the ohmic resistance of the circuit. In familiar

symbols, C being determined so $i = o$, when the circuit is first completed, we have

$$i = \frac{E}{R}\left(1 - e^{-\frac{Rt}{L}}\right) \qquad (20)$$

It is not necessary in this group of phenomena to consider α as a derived element only. It may be a primary element, in which case Ω will be a "coefficient of restraint."

Thus according to the arrangement shown in Fig. 4, let it be required to find the variation with the time, μ, of the charge, α, in the condenser of capacity, C_1, when the constant E.M.F., F_1, is

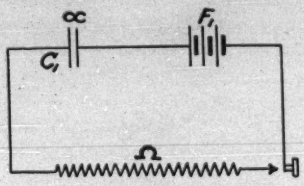


Fig. 4

suddenly applied through the high resistance, Ω, to the terminals of the condenser. If ν is the instantaneous potential to which the condenser is charged and $i = \frac{d\alpha}{d\mu}$, is the instantaneous current in the resistance, Ω, we have the relation,

$$\nu = \frac{\alpha}{C_1} \text{ and } \nu = F_1 - \omega i$$

or

$$\omega \frac{d\alpha}{d\mu} = F_1 - \frac{\alpha}{C_1} \qquad (21)$$

Now Eq. (21) is observed to be exactly like Eq. (16) where $K = \frac{1}{C_1}$. The charge, α, is a primary element, as a quantity of electricity is a fundamental concept, and Ω is a "coefficient of restraint," as it is the constant factor which determines the rate of change of the element α. Equation (21) integrated, expressed in familiar symbols, and the constant determined so that the charge is zero when the time is zero, becomes,

$$q = E C_1 \left(1 - e^{-\frac{t}{R C_1}} \right) \qquad (22)$$

Equations (20) and (22) are evidently perfectly analogous, though in the first the phenomenal element, that varies, is a derived element and in the second it is a primary element.

Groups of phenomena, allied to the above might be expressed by assuming that the passive cause of equation (16) instead of being directly proportional to the characteristic element, is proportional to any function of this element. The function, α^2, is one which occurs frequently, the damping effect then being usually produced by a medium such as air.

In another important group of phenomena the effect is manifested under the following conditions: The active cause which initiated the effect has ceased to act, and the characteristic element, α, continues to exist only in virtue of the object, in which the effect is manifested, being possessed of inertia. α is then a derived element, Ω a coefficient of inertia, and equation (2) holds. Further, we have the variation of the derived element, α, produced by the simultaneous action of a passive cause, which we shall take proportional to α, and by an active cause which has its origin in the object or system itself. This active cause will be taken proportional to the primary element, ϵ. As the active cause external to the object has ceased to act, F_1 of equation (1) will be zero. The passive cause has the nature of a frictional resistance and the work that is done against it by the inertia of the system is converted into heat and finally lost to the system. It is then, as in Equation (16), equal to $-K\alpha$ and will correspond to F_2 of Equation (1). The internal active cause will act first in a direction opposite to the external active cause while it initiates the effect, and hence must be given a negative sign. As it is taken here proportional to the primary element it equals, $-K_1 \epsilon$, and corresponds to F_3 of Equation (1). With these assumptions the group of phenomena is expressed when Equation (1) assumes the form,

$$\omega \frac{d\alpha}{d\mu} = -K\alpha - K_1 \varepsilon \qquad (23)$$

and this in conjunction with Equation (2) becomes,

$$\omega \frac{d^2 \varepsilon}{d\mu^2} = -K \frac{d\varepsilon}{d\mu} - K_1 \varepsilon \qquad (24)$$

If Eq. (23) be differentiated in respect to d μ and $-\frac{d\epsilon}{d\mu}$ replaced by its value α, derived from Eq (2), we have

$$\omega \frac{d^2\alpha}{d\mu^2} = -K \frac{d\alpha}{d\mu} - K_1 \alpha \qquad (25)$$

As equations (24) and (25) are of like form, they show that the primary element, ϵ, and the derived element, α, follow the same law in their variation. The integrals of these equations can therefore only differ in their arbitrary constants. The primary element will be of the nature of a space coördinate or a quantity of electricity, while the derived element will be of the nature of a velocity or a current of electricity. The quantity, d μ, will signify an infinitesimal increment in time.

The equations (24) and (25) are recognized as symbolizing the motions of masses and electricity in a very well known group of analogous phenomena. From this group we may cite the following: The rectilinear motion of a mass attached to a spring, the motion being resisted by some frictional force which is proportional to the velocity; the motion of a pendulum bob making small swings in a medium or magnetic field that damps its motion proportionately to its velocity; the motion of a D'Arsonval galvanometer system which is magnetically damped; the movement of a liquid between two tanks joined by a pipe, the liquid standing higher in the one tank than in the other when communication between the tanks is first established; the movement of electricity between the coatings of a leyden jar, the circuit being completed through a self induction and an ohmic resistance. In the cases of motions of masses the solutions of equations (24) and (25) give space coördinates and velocities, and in the electrical case quantity of electricity and electric current. The relations of corresponding elements of the phenomena cited might be put in evidence by a table or briefly stated in part as follows: For the pendulum, Ω, is a quantity which equals the mass of the bob, α is the angular velocity of the pendulum, ϵ the angular displacement from the vertical at any time, μ; K is a constant quantity that determines the extent of the damping which otherwise depends upon the velocity and K_1, a constant which depends upon the weight of the bob.

In the movement of a galvanometer system, Ω is the moment of

inertia of the coil, α its angular velocity, ϵ the angle through which it is displaced at any instant, μ; K is a constant which depends upon the damping power of the magnetic field, and K_1 a constant which depends upon the torsional strength of the fibre. In the electrical case, Ω, would be a coefficient of self induction, α, the instantaneous value of the current in the circuit, ϵ the quantity of electricity in the condenser, at any instant μ; K the ohmic resistance of the circuit and K_1, the reciprocal of a capacity.

The solutions of Equations (24) and (25) and the discussion of the many interesting features of the group of phenomena that they symbolize are too lengthy for the limits of this article. The curves (a), (b), (c), and (d), shown in Fig. 5, will illustrate the

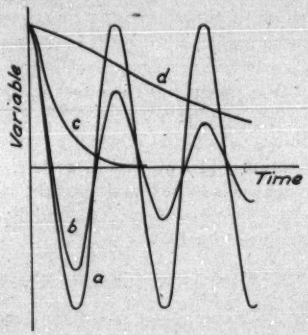


Fig. 5

four characteristic features to which the solution of the equations lead. The curve, (a) exhibits undamped oscillations, in which case the constant K is zero. This case can only, of course, be approximated by natural phenomena. The curve (b) shows a dying-out oscillation, which is the case most ordinarily encountered in nature. The curve (c) shows the primary element, ϵ, returning to a zero value by so-called aperiodic motion. In the case of a galvanometer, the system is then said to be critically damped, and the conditions are filled for the quickest possible return to zero. The curve (d) illustrates the case in which the constant K is large and the system is over-damped.

The reader interested in following in detail the developments suggested by the study of equations (24) and (25) is referred to: "Alternating Currents," by Bedell and Crehore, chapter VII, and "Instruments and Méthodes de Mesures Électrique," par H. Armagnat, chapter I.

Enough cases of the transformations and applications of Equation (1) have now been given to fully exemplify how, by proceeding to more and more extensive modifications of this equation, nearly the whole realm of physical phenomena might be symbolized. Not only will equation (1) in conjunction with equation (2) express many different groups of phenomena; it will also allow a kind of classification to be made of the groups themselves as well as of the phenomena in each group. There is one further extension of equation (1) that an account of its importance and general applicability must be mentioned but which the limits of this paper will not permit us in any way to develop. Thus, the active cause producing the effect in very many, if not most phenomena, does not cease to act after the effect has been initiated but continues to act, and is usually some function of the time. Then in equation (1) we should write $F_1 = \varphi(\mu)$, to signify that the active cause is some function of the independent variable If also we have $F_2 = -K_1 \varphi_2(\alpha)$ and $F_3 = -K \varphi_3(\epsilon)$, equation (1) assumes the form,—

$$\Omega \frac{d\alpha}{d\mu} = \varphi_1(\mu) - K_1 \varphi_2(\alpha) - K \varphi_3(\epsilon) \qquad (26)$$

By the relation $\alpha\, d\mu - d\epsilon = 0$, equation (26) becomes,

$$\omega \frac{d^2\epsilon}{d\mu^2} = \varphi_1(\mu) - K_1 \varphi_2\left(\frac{d\epsilon}{d\mu}\right) - K \varphi_3(\epsilon) \qquad (27)$$

Equation (27) may be integrated when

$$\varphi_2\left(\frac{d\epsilon}{d\mu}\right) = \frac{d\epsilon}{d\mu}, \quad \varphi_3(\epsilon) = \epsilon$$

and, $\varphi_1(\mu) = E \sin 2\pi N \mu$ where N is the frequency, or the number of times in a unit time that $\varphi_1(\mu)$ assumes the same value. The equation now takes the familiar and important form,

$$\frac{d^2\epsilon}{d\mu^2} = \frac{E}{\omega} \sin 2\pi N \mu - \frac{K_1}{\omega} \frac{d\epsilon}{d\mu} - \frac{K}{\omega} \epsilon \qquad (28)$$

If E is an electromotive force, ϵ, a quantity of electricity, Ω, a coefficient of self induction, K_1, an ohmic resistance and K the

reciprocal of a capacity the equation expresses the law of the movement of electricity in a circuit having inductance, resistance, and capacity, when a harmonic E.M.F. is applied at its terminals. The complete discussion of the solution of equation (28), so interpreted, is to be found in Bedell and Crehore's "Alternating Currents," chapter VI and following. The point to emphasize, is that the equation should not be thought of as being applicable to the electrical case only. It expresses existing relations between causes and their effects not only for electricity in a circuit but for a group of perfectly analogous phenomena. The student of mathematical physics should consider that, when he has understood these relations for one kind of phenomenon, he is equipped for seeing them in all the phenomena of a group. For example, he should find his knowledge of the equation equally applicable to a study of the movement of the system of an oscillograph, the system being partially damped, and kept in motion by the application of a periodic E.M.F. of sine wave form. Thus, if the same E.M.F. be simultaneously applied to the system of a second oscillograph, which has its system so damped as to correctly register the applied E.M.F., a certain phase difference will be shown between the traces made by the two instruments. If an electric circuit has its corresponding constants related in like manner to the constants that govern the movement of the system of the first oscillograph, then similar phase relations should maintain between E.M.F. at its terminals and the current flowing, as maintains between the traces made by the two oscillographs. Such extension of the concepts to the various phenomena of the group results most profitably in breadth of view, and depth of insight into physical relations.

Equation (1) is a general expression that may be made to symbolize the relations which exist between an effect and the cause or causes to which the effect is due. Now, the phenomenon, when expressed by the equation, is the change which occurs in an element, α, chosen to characterize the phenomenon, when a change takes place in an independent variable, μ. The equation implies that a uniform variation is taking place in the arbitrarily chosen quantity, μ, which may designate a time, a space, or other independently chosen variable. But there exist numerous geometrical relations, facts of experience, or quantities of definition that bear fixed relations to other like relations, facts, or quantities,

with which the idea of change in the elements characterizing them with time or space or other independently varying quantity, is not in any wise connected.

A consideration of such fixed relations in the above cases may often reveal illuminating analogies. Thus, if C is the circumference of a circle, D its diameter, and A its area, the fixed geometrical relation maintains that $\frac{C}{A} = \frac{4}{D}$ and, if V is the volume of a sphere of the same diameter, and S its area, then $\frac{S}{V} = \frac{6}{D}$ As these two relations are given by expressions of the same form an analogy may be said to exist between them. Another example of an analogy between geometrical relations is given by the diagram below, which explains itself.

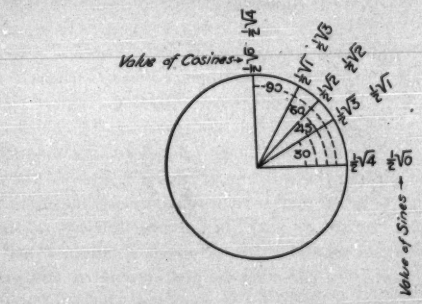


Fig. 6

The analogy consists in the values of the sines being the same as the values of the cosines, if the order in which one is read be reversed. The perception of this and similar symmetrical relations derives its value chiefly from the assistance given the memory, though it may in some cases lead to a deeper insight which will reveal unsuspected and useful relations.

As a fact of experience it is noted that two electric charges of unlike sign Q_1 and Q_2 attract each other by the law,

$$F_1 = - K_1 \frac{Q_1 Q_2}{d^2},$$

and likewise that two masses M_1 and M_2 attract each other by the same law which is given by the expression:

$$F_2 = K_2 \frac{M_1 M_2}{d^2},$$

hence it may be said that a perfect analogy exists between these two facts of observation, though no notion of a change with time is necessarily involved.

Or again, adopting the definitions that are given to the two quantities, electrostatic capacity and electrical conductance, a relation is found to exist, in any particular case, between the expressions for each which shows that a perfect analogy exists between these two quantities of definition.

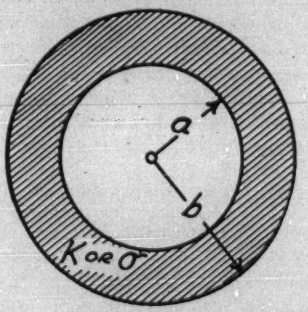


Fig. 7

An example will show this: Take two concentric spherical surfaces (Fig. 7), the inner one of radius *a*, and the outer one of radius *b*, and fill the space between them first, with a dielectric of specific inductive capacity, K. It may be easily shown (See Elements of Electricity and Magnetism, by J. J. Thomson, Ps. 86 and 136) that the expression for the capacity of the condenser so formed is,

$$C = \frac{K\,a\,b}{b - a} \qquad (29)$$

Second, fill the space between them with an electrical conducting material of specific conductivity, σ, and pass current from the surface of one sphere to the surface of the other. We shall then

have as the expression for the electric conductance, or reciprocal of the resistance, of the space between the spheres.*

$$G = \frac{4\pi\sigma a b}{b - a} \qquad (30)$$

As expressions (29) and (30) are of the same form, a perfect analogy exists in this case between capacity and conductance. Indeed, it may be shown that in general the relation holds that $\frac{G}{C} = \frac{4\pi\sigma}{K}$. The same analogy might be extended to heat conductance and other cases and many interesting deductions be drawn, but enough has been given to illustrate the character of analogies based upon geometrical relations, facts of experience, and quantities of definition. In none of these cases is the notion of change involved as an independent variable changes and hence these analogies would group themselves apart from those which are governed by equation (1) in which effects, as changes in a characteristic element, are manifested as the result of causes acting to produce them.

ANALOGIES REGARDED IN THEIR PHYSICAL ASPECTS.

The popular conception of what constitutes analogies, in physical science, is certainly not that of a likeness between the forms of different mathematical expressions, such as has been illustrated in what precedes. The analogies commonly used, and the views of them ordinarily held, are physical rather than mathematical in character. If certain physical relations or phenomena are shown to have characteristics in common with certain other, and usually more familiar, physical relations or phenomena, an analogy is said to exist between them. If a model is constructed or pictured, which in its form or movements imitates certain physical occurrences, the former is called an analogue of the latter.

In agreement with this conception of analogy we find numberless instances, in text-books, treatises, lectures and common speech, of use being made of one form of familiar phenomenon to *illustrate*, and the attempt often made to *explain*, others that are less clear or less familiar. Physicists of eminence have made

*See Ibid, p. 317.

frequent use of analogies, treated physically, and have employed mechanical models, as analogues of phenomena, to assist their imagination and to aid them to present their concepts with greater clearness. Thus, the late Lord Kelvin has said in effect:

"It appears to me that the true meaning of the question: Can we or can we not understand a particular subject in physics? is this: Can we make a mechanical model to correspond? I am never satisfied until I have been able to make a mechanical model of the object; if I can make a mechanical model I understand; as long as I am not able to make a mechanical model, I do not understand."

Faraday's lines of force conceived as elastic threads, that mutually repel transversely, is an analogue of electric and magnetic strains in the ether that has been universally used with most fruitful results. Lodge, in his "Modern Views of Electricity," has developed with much effect this analogy, as well as many other hydrodynamical and mechanical analogues. Maxwell, a master in mathematical physics, made not infrequent use of mechanical analogues and analogies. The properties of a dielectric are greatly illuminated by his mechanical illustration described in section 334, Vol. I, Electricity and Magnetism. J. J. Thomson, another analyst of the greatest power, makes frequent use of analogy, to illustrate his physical concepts. See for example his analogue of the instability of electrical equilibrium of two equal, adjacent atoms.*

The employment of such analogies as those cited, when they are not accompanied by mathematical treatment, have usually been used to assist the mind to rapidly review and grasp in outline the leading and essential features of a group of phenomena. They are not commonly employed in such cases to furnish quantitative results. But the mathematical relations between the characteristic elements of a group of physical phenomena, that are referred for clearness of illustration to a concrete analogue, may also be given. The phenomena of the group will then become joined together mathematically and *illustrated* physically. We shall now endeavor to show that, by the extensive use of analogy, in which the characteristic elements of phenomena are quantitatively related by mathematical expressions, and illustrated by

*Corpuscular Theory of Matter, p. 128, by J. J. Thomson.

concrete representations of the elements, the greatest practical ends are to be obtained. These practical ends are to be found in vividness of perception, clearness of insight, rapidity in generalization, economy of thought, and fertility in conception. It must, however, be remembered that, as the French say, "Comparaison n'est pas raison." To compare a wave motion in the ether to a mechanical model, is not to give a physical theory of such wave motion, "because it lacks that which is indeed the essence of a theory, the unity which connects in a rigorous order the laws of various groups of phenomena." But analogies and analogues, if they do not constitute physical theories, guide the mind toward framing them, and they serve beside practical ends of the greatest import. Now it is evident that the possibility of perceiving the existence of analogies between our physical concepts of nature implies that these concepts are present in the mind. This in turn, implies an acquaintance more or less intimate with phenomena themselves.

If there is familiarity with one class of phenomena, as those of the movements of matter and the conditions for its equilibrium, then a less familiar group, as the electrical phenomena, may be more quickly and easily grasped when the analogies between them become revealed either by instruction or original observation. It is an economy in mental effort, and an assistance to a well ordered memory when the corresponding elements of analogous phenomena are perceived. Thus in trying to form correct notions of the propagation of radiant energy, as light, heat, or electricity, it is helpful to refer what is analogous to the more familiar phenomena of the propagation of sound.

The ability to develop from a few primary concepts, a system of mathematical relations which are susceptible of expressing the relations of the elements of many phenomena, will not relieve one from acquiring an acquaintance with the facts, if the phenomena are to be truly understood. It is an illusive hope to suppose that because one can use physical mathematics with skill, therefore, he can know physics, without physical experience.

The above statement is thought so profoundly important that we shall attempt at some length to illustrate its force by a concrete and convincing example: For this purpose use may be made of the analogy found above, between a cylinder filled with mercury and carrying a current I, parallel to its axis, and

the same cylinder rotating on its axis with angular velocity, ω. The mathematician makes certain assumptions respecting these two cases. He mentally abstracts from the two cases the phenomena P_2 and P_3 and finds that a perfect analogy exists between the two which has been given above in equations (k) and (l) by the expressions $g_2 = -\pi I_1^2 R^2$ and

$$g_3 = \frac{\rho}{2} \omega^2 R^2.$$

These expressions tell him that in the first case, when current of density I_1, is flowing through the cylinder in the direction of its axis, that a difference in hydrostatic pressure is produced between the circumference and the axis, and that this pressure acts from the circumference toward the axis, and in the second case that, if the cylinder revolves on its axis with angular velocity, ω, a difference in hydrostatic pressure is produced between the axis and the circumference and that this pressure acts radially outward. The expressions give also the numerical values of these differences in pressure in the two cases. Now, unless further assumptions are made equations (k) and (l) give no more essential information.

Let us now see how a physicist, fully acquainted with the physical facts, could treat the problem, and by following along steps that are naturally suggested by the physical considerations of the case, arrive at interesting and, possibly useful results For this purpose let us design on paper a model that would be actually operative. Such a model is shown in Fig. 8, A and B.

B shows the model in vertical section and A, a section on the line a—b of B. The drawings have been made, complete in detail and the parts fully designated so as to save a detailed description. Now the model, as drawn, shows three compartments, which are filled with mercury. Compartment (1) at the top opens into a central tube T, and compartment (3) at the bottom opens into two tubes, t_1 and t_2, parallel to the axis of the cylinder. The entire solid portion of the cylinder is constructed of copper except the containing walls, which are made of a metal coated on the inside with an insulating enamel. The compartments (1) (2), and (2) (3) are connected by tubes having a thin lining of insulating material, and pass, as shown, from the circumference of (1) to the center of (2) and from the circumference of (2) to

4 of

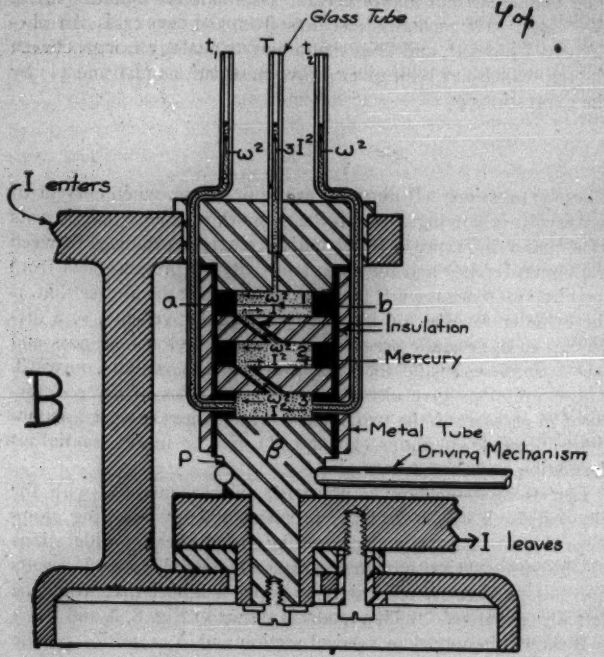

Glass Tube
t_1
T
t_2
ω^2
$3I^2$
ω^2
I enters
a
b
Insulation
Mercury
B
Metal Tube
Driving Mechanism
p
I leaves
Section on a-b.
Metal Tube
Tube
Insulation
Space for Mercury
A
Tube

Fig. 8

the center of (3). By the pulley, P, the cylinder may be made to rotate with an angular velocity, ω. When the cylinder is stationary, or when it is in rotation, a current of density, I_1, direct or alternating, can be passed, in an axial direction, through the cylinder, passing through the mercury in the compartments and through the copper partitions which separate them, except where the small connecting tubes go through them.

First suppose the cylinder is stationary and that a current of I_1 amperes per square centimeter is passed through it. The equation (k) tells us that a lower hydrostatic pressure will exist at the circumference than exists at the axis, and we should therefore expect the mercury, as in fact it would, to rise in the tube, T, and depress in the tubes, t_1 and t_2.

But here is what the equation does not tell us, and which we must be led to find out by considering the problem physically, namely, that the pressure difference produced in compartment (3) is added to the pressure difference in compartment (2) and this in turn to that in compartment (1). Hence the mercury will rise three times as high in the tube, T, as the equation (k) would indicate.

Second, suppose the current is not passing, but that the cylinder is rotated with an angular velocity, ω. Equation (l) tells us that the mercury will lower in tube T, and rise in the tubes t_1 and t_2. But, by physically conceiving this case, we see that no more than one compartment can be effective in producing a pressure-difference. This is because the pressures set up are hydrostatic, and however large or small we make the diameters of the slanting tubes connecting the different compartments, the centrifugal force acts in these tubes, as well as in the compartments, thus preventing the pressure difference of one compartment being added to that of an adjacent compartment.

In this respect then, namely, of the pressures being added in the one case and not in the other, equations (k) and (l) are not analogous and on this point suggest no information. We are taught, however, by the physics of this case that we must introduce in the first instance a factor, N, that expresses the number of compartments that are joined by the slanting tubes in series. Equation (k) then becomes,

$$g_2 = -\pi N I_1^2 R^2 \qquad (31)$$

Or if we write I for the total current passing through the cylinder, we have, since $I = \pi R^2 I_1$

$$g_2 = -\frac{N I^2}{\pi R^2} \tag{32}$$

We are now led to note that by applying the right current and the right angular velocity, the oppositely acting forces produced by the current and by the rotation may be made to balance. The balance will be indicated by the mercury in the tube, T, showing no change of level when current is passing and when the cylinder is rotating.

By writing the equation denoting the condition of balance, *i. e.*,

$$\frac{\rho}{2} R^2 \omega^2 - \frac{N I^2}{\pi R^2} = 0 \tag{33}$$

we obtain as the value of the current

$$I = R^2 \omega \sqrt{\frac{\pi \rho}{2 N}} \tag{34}$$

It is also possible to obtain the value of the current independently of the radius, R, of the cylinder. To do this, stop the rotation but keep the current unchanged. The mercury in the tube, T, will then rise a certain height, h, above the level of the mercury in tubes t_1 and t_2. If g is the intensity of gravity, we shall have,

$$h = \frac{g_2}{g \rho} = \frac{N I^2}{g \rho \pi R^2} \tag{35}$$

Now, R_2, may be eliminated from equations (34) and (35) giving

$$I = \frac{h g}{\omega} \sqrt{\frac{2 \pi \rho}{N}} \tag{36}$$

This unusual method of obtaining in absolute measure the value of a direct or an alternating current in terms of a height, the intensity of gravity, an angular velocity and a density, was led up to very naturally by considering a particular case in its physical aspects and with a knowledge of the physical facts involved. The mathematician attempting to build up a system of analogies, by starting with a few fundamental concepts and from these developing systems of corresponding equations would scarcely arrive at a result like the above. The physicist, however, fully ac-

quainted with physical facts and using his mathematics as a tool and not as a guide, is very naturally conducted by the suggestiveness of the object studied to new analogies and new points of view. Moreover, his acquaintance with physical facts teaches him to interpret his equations of analogous phenomena at their true value as expressive of real physical law. Thus, in referring to equations (24) and (25) a well informed physicist would interpret these equations, when applied to the electrical case of electricity oscillating in a circuit of capacity, inductance, and ohmic resistance, as very approximate only. He would know what the mathematical view, alone, would not, and in fact did not, teach, that the resistance of the circuit is a function of the frequency of the oscillation, and may not, therefore, as assumed in the equations, be treated as a constant of the circuit.

In tracing and building up analogies by conceiving phenomena physically and by forming mental images of concrete illustrative examples of the phenomena, the *like elements* common to the phenomena of a group, and the *unlike elements* not common to the phenomena of the group, are both revealed. It is the like elements that determine the analogies if they exist, and by these elements the phenomena are classified and grouped together. But it is the perception of the unlike elements that leads to new points of view, to new relations and to the forming of new analogies. The perception of the contrast between the like and the unlike will lead to inquiry as to wherein the difference lies. Thus experiment and further investigation are suggested and the path is opened to enlargement of view and possible discovery. In expressing the relations between the elements of phenomena by mathematical equations, only those elements are dealt with which were mentally abstracted in the original assumptions. If then mental images of concrete cases are not before the view, no new elements will come into the combinations formed; new points of view will not be obtained, and invention is unlikely, and discovery is impossible to follow. The mathematician who depends upon his symbols to give him new results is very like a child with a kaleidoscope. The child following his fancy may take from a pile of bits of colored glass a number of pieces that attract him. If then he places them in his kaleidoscope he thinks perhaps to find something new by watching the endless combinations of symmetrical forms. But if he did not chance to select red, or green, then these colors

will never appear to him, however long he turns the combinations. To get red or green he must return to the pile of colored glasses, the only source of new matter. So the mathematician who symbolizes his original assumptions, and then neglects the source from whence they come, will never find anything new however long he studies their fascinating combinations. To find what is new, and indeed, to increase the possibility of the combinations, he, too, must return to nature.

When two or more objects or phenomena are presented, in concrete form, it is almost always possible to find an analogy between them in respect to some of their features or some of the elements that attach to them. Now these elements, taken as a basis of comparison, may be merely accidental elements, as of form, color, temperature, and the like, and not elements that are characteristic of the object or phenomenon; that is, elements that must be considered in framing an exact definition of the object or phenomenon. Where analogies are traced that depend upon elements that accidentally attach to the particular objects in view, they are lacking in value from the standpoint of science. As the tracing of accidental and unexpected relations between non-essential features belonging to objects, often gives pleasure, and new points of view, it has a legitimately recognized value for literature. Thus to say,—

> "The quality of mercy is not strain'd,
> It droppeth as the gentle rain from Heaven,
> Upon the place beneath;"

is to trace an analogy that has fine merit in literature but is worthless in science. Such an analogy cannot help to extend or clarify scientific concepts. In fact, in the early development of science, the use of such loose analogies did great harm. They draw the attention from elements that are essential in defining objects, to elements of a non-essential and accidental character.

In referring phenomena to a phenomenon with which there is a perfect analogy, in the true scientific sense, the phenomenon selected as the standard phenomenon, so to speak, should be one capable of very precise definition, and one that is as far as possible universally and instinctively comprehended.

In attempting to unify physics and to arrange its varied phenomena in some kind of order and to trace among these phenom-

ena as many likenesses and connecting links as possible it should be enquired what classes of phenomena are the most universally and instinctively known. These phenomena will be those best suited to serve as prototypes of the different groups of phenomena. The phenomena in question are undoubtedly those of the equilibrium and the movements of ordinary solid matter. In other words the phenomena of mechanics are those that are most precisely defined, the best known formally as well as instinctively, and may best serve as prototypes of all physical phenomena. We are thus led to briefly examine the rôle played by mechanics in the scientific analogies that exist among physical phenomena.

THE RÔLE OF MECHANICS.

In accordance with what we have shown in the beginning of this paper it is evident that, if any phenomenon can be expressed by an equation of the same form as one that expresses a mechanical phenomenon—that is the relation between the movements of a material system and the forces that act upon it,—the two are analogous. Throughout the later developments of physical science, puissant attempts have been made to *explain* all physical phenomena, including those of heat, on a purely mechanical basis. But even with the assumptions that have been made by Hertz and others of the existence of "concealed masses, concealed motions, and concealed cöordinates,"* this attempt appears to the writer little likely to become successful.† But, if we limit the use of the movements of mechanical models and systems, to *illustrate* all the features of phenomena that are capable of interpretation and of being understood, then we may turn to mechanics as furnishing models par excellence of all physical phenomena. Mechanics, in this interpretation, in addition to its fundamental character, may serve to play a most essential rôle by associating together and cöordinating in the mind the whole domain of physical manifestations. From infancy, and indeed by phylogenic experiences from an unknown past man has acquired an instinctive and unerr-

*The Principles of Mechanics by Hertz, Section 595, and Introduction, pages 25 and 26.

†Consult on this important matter L'Evolution de la Mécanique, Chap. XV, par P. Duhem.

ing perception of mechanical laws. If he cannot formulate them he can always adjust himself to them. In the domain of light, of sound, and especially of electricity, it is not so. The phenomena must here be learned under specially arranged conditions. When, therefore, one begins to formulate his concepts of the invisible, and intangible manifestations of electricity or of the wave motions of radiant energy, it is a powerful and proper instinct that leads him to visualize in terms of mechanical things he knows, the new phenomena that he strives to comprehend. By his mechanical concepts it becomes possible for him to give a concrete aspect to what otherwise may remain a vague abstraction. From this point of view, the mental or actual construction of mechanical models, that are capable of movements which are related to time and to each other in the same way that the variations in the variable elements of phenomena, considered non-mechanical, are related, serve an important purpose and satisfy an intellectual necessity. Even a mind that is capable of powerful abstraction must lay hold of something. It may be conventional graphical representations, or it may be mathematical symbols, and, it is true, many will contend that these last will serve as well as the more concrete and specialized mechanical models. These would reason from an individual case as follows: The sinusoidal feature of an alternating current may be represented in many, if not all, of its essentials, by a model. This model may consist of a long row of small white balls attached to the ends of vertical wires that by their properly timed up and down movements cause the balls to show a progressive wave motion of sinusoidal character. Now, it may be truly said, that this model is as unlike the electricity that moves as would be a system of lines drawn on paper to represent the wave motion. The model by its movement represents progressive variations, with the time, like the variations in the elements of the phenomenon that it represents. But systems of lines on paper may be made equally to show these variations. It is only necessary to represent the successive stages of the motion and make use of a stroboscopic apparatus to make the lines do exactly as well as the model. But why not write the equation,

$$y = I \sin (\varphi + 2 \pi N t).$$

Does it not tell us exactly as much as the model or the diagrams,

and is it any more unlike electricity than balls on wires and lines on paper?

As neither model nor lines, or equation *explain* the phenomenon, but only represent the variations in those elements of the phenomenon that one has chosen to fasten his attention upon, why not use the equation as the most compact, cheapest, and manageable symbol of the reality? Nevertheless, we believe there is a great value in the model, though we would not contend that it shall *replace* the equation. We do contend that its value is such that it should *supplement* the equation and for these reasons: In viewing a model or a mental image of one, all the essential relations, respecting the phenomenon, that it expresses, can be taken in, so to speak, at a glance, and viewed in their entirety. In consequence of this a vivid and lasting impression is made upon the memory, which the mind often will have occasion to recall when analogous phenomena to those represented are brought under contemplation. Again, the model makes its strong appeal to the universal, instinctive knowledge of our race, while the equation appeals to a highly specialized knowledge that many excellent minds never acquire. But most important of all is the suggestive character of the model. The equation was formed out of elements deliberately abstracted from the phenomenon, and once having been formulated, all other possible elements, that might have been considered, are henceforth totally neglected and forgotten. The equation can give all possible combinations of the elements originally abstracted and that is all. Not so with the model. In observing its form and in noting its movements, new characteristics other than those first contemplated, are seen, and these new elements being noted in the model, the mind with great naturalness enquires if these new elements may not also belong to that which the model represents in other respects. Thus further examination is suggested, and invention and fertility follow.

We should, however, not assume other values for our models and our analogies than one of a purely practical character. The tracing of the analogies and the concrete representations of them has not given us a law or formulated a theory. In other words, it has not *explained* the phenomena in a broad sense. But this practical gain is most important. The vast majority of our judgments passed upon the relations that maintain between causes and their effects when presented to us, must be made not by criti-

cal analysis, but by referring the physical event in question to some analogous event with which the mind has already been familiarized. The greater our possession of true mental images, and the greater the number of these images that are joined together in natural analogous relations, the more swift and unerring are sure to be our judgments of new phenomenal relations. Until, however, these judgments are given the precise setting of numerical relations, they must remain qualitative rather than quantitative. And herein lies the necessity in the exact sciences, of being able to formulate in the precise language of mathematics, the relations between the phenomenal elements that we select for consideration. The physicist, with strongly developed visualizing power, and a broad view of analogous relations, is not therefore free to depart from the employment of mathematics and the methods that it teaches. Nor, on the other hand, may we hope to wholly rely on this powerful instrument of thought, if we would frame concepts of the external world that may fairly represent it.

If we now return to the developments which we made of equation (1) above, and consider the innumerable further extensions that a mathematician might make of it, we may see how barren would be the results obtained for increasing our knowledge of nature, unless parallel with these developments, a physical illustration and a physical interpretation be carried along.

ASSOCIATIVE PHYSICS.

The science of physics is studied to know nature, to instruct others, and to serve the other sciences and engineering toward practical developments. One or all of these ends may be the objects of its pursuit, but we may enquire if the same methods in its study are not called for in any case? Now it seems evident, that with any of these ends in view, it is equally important to the student of the science to observe the best mental economy whereby his powers may be conserved to compass as far as possible the vast extent of his subject, to acquire the most broad and broadening view, that the study may serve as a culture to his mind and enlarge instead of narrow his general scope, and finally, to furnish him with a strong command of the largest possible number of essential facts. All these demands may best be met,

for whatever ends his physical science is pursued, if he observes a fundamental psychological law of this character; that new knowledge is only acquired by association of the ideas formed through new perceptions with the concepts that are already firmly possessed. As ideas are associated, so is a mind constituted. In one mind, bound with the iron chain of habit, the ideas are of a mosaic character. The characteristic elements and the natural connections of phenomena and objects are unnoted because the accidental and not the fundamental elements have been habitually fastened upon. But in a better type of mind, the associations between the ideas belong to those characteristic elements that define objects and phenomena. The principles and the laws that govern large groups of phenomena are mentally associated in a harmonious arrangement. When a new perception comes, it finds forthwith its orderly setting among extensively connected concepts already possessed. The true value and relationship of the new perception is quickly and intuitively estimated and the remembrance of the perception only retained if it is worth while. If in this mind the new perception is seen to have no natural connection with what is already possessed, intense curiosity is immediately aroused and investigation is started; and so come discoveries.

To the first type of mind a multitude of particulars, that lack logical connection, must burden and overwhelm the most vigorous intellect, if it tries to gain a grasp of the vast domain of natural phenomena.

To the second type of mind, made by habit, wisely selective in its judgments and memory, the essentials only, that characterize large groups of particulars, are given consideration. This mind, with perhaps less native intellectual vigor than the other, can grasp a far larger universe and better impart the knowledge of it to others, and make of this knowledge more extensive use for practical ends.

To help toward the formation of such a well ordered and economically working mind physical science should be viewed, taught and used as an association of facts, laws, principles, and manifestations that appear similarly and indifferently in all its, at present, artificially separated branches.

For the most part, modern general treatises on physics start out with an exposition of the principles of mechanics, then fol-

low treatments of the properties of matter, sound, light, static and dynamical electricity, and magnetism, followed perhaps with some added chapters upon radiant electric energy, Röentgen and Becquerel rays and the recent discoveries in radio-activity. Little or nothing is taught respecting the intimate relationship and the profound analogies between all these subjects. Seldom is a word spoken concerning the bearing of all the isolated facts and principles exposed, upon the intellectual and practical needs of man. The profoundly important philosophy of the science is entirely omitted. Each subject is set forth to be learned and, alas, memorized, by the student as so many distinct and separate branches of physical science. Perchance in riper years, if the student pursues his studies, and after the formal and piece-meal knowledge that he early acquired is forgotten, he will come to realize, and delight in the thought, that a true and broad physics is not sets of equations, nor measurements of lengths, volumes, and curvatures, but a philosophy that applies to human life. He will see that its subject matter reaches to the confines of the universe and has relationship with every interest known to man. He will perceive that its grand laws and principles simply express in orderly form our most intimate knowledge of the world we live in. And he will see, too, that after all, "the most sublime scientific knowledge, in the last analysis, has no other foundation than common sense."

INCREASING USE OF STORAGE BATTERIES.

It is predicted by the *Electrical World* that the use of storage batteries will increase greatly in the next few years. It cites the belief of many engineers that the steam turbine marks only a transition period in prime movers, which must give way to the internal combustion engine because of the superior economy of the latter. The gas engine has no overload capacity; it operates best when delivering a steady amount of power, and is liable to give trouble when worked on light loads. The storage battery affords the required flexibility in respect to variations in the external load while maintaining a constant load on the generating equipment; as one writer puts it, "the storage battery is the natural consort of the gas engine." An extended use of gas engines for driving electric generators will undoubtedly be attended by a corresponding increase in storage battery work.

THE FRANKLIN INSTITUTE.

The Equipment of Farms and Country Houses With Electricity.

By Putnam A. Bates, New York, N. Y.

The difficulty of obtaining satisfactory farm hands and of providing for them makes the machine, which can be substituted for manual labor, a decided advantage to the land owner. This fact, together with the greatly improved quality of illumination and convenience which electricity gives, is causing a growing demand for a reliable and reasonably economical source of electrical energy with which to supply both light and power on the larger country estates and farms.

The advantages of convenience and cleanliness with the use of electricity are pretty well known and appreciated, but the means by which electric currents may be obtained, economically, and how this form of energy may be applied to bring about sufficiently better returns to justify the installation of an isolated plant to produce it, are points not so generally understood.

A brief resume, therefore, of what can be done and what has been done in this direction may serve of some interest to those who have chosen farming for their pleasure or their livelihood.

Within the last few years the small internal combustion engine and the electric storage battery for stationary service have been so much improved and simplified as to cause them to compare very favorably with the better known types of power producing apparatus in reliability of operation and in first cost. The extreme simplicity of both this type of engine and of the storage battery, together with the very great economy in fuel consumption of these engines, the low price of fuel, and the efficiency of the battery as a device for storing the energy and delivering it in the form of electric current when needed and in the quantity required, result in a very low operating cost.

The advent of tax-free alcohol in the field of available fuels for use in internal combustion engines is being watched with interest by those concerned in this problem, and while it is too early as yet to make any predictions, this fuel has already given promise of, in time, becoming a strong competitor of kerosene and gasolene in this field.

There is still another promising source of power for these small plants which is too often overlooked. That is, the small water powers, which are scattered through the country, most of which could be easily developed at small expense, and many of which have already been developed for commercial purposes, abandoned because of changed conditions, and require simply the addition of the electrical apparatus to make them valuable sources of energy.

Such a water power plant could frequently be made to serve a group of users of electric current at very small first cost for each individual, and at an operating cost per kilowatt hour which would be almost nil.

It is, therefore, evident that the means are at hand by which those not reached by a reliable and low-priced electric service may still avail themselves of the electric current and obtain the many conveniences which it contributes, at a cost which should seldom exceed that of an equivalent service from a public service electric plant at the rates ordinarily prevailing, for such service, throughout the country.

It must not be lost sight of, however, that very much depends upon the design of these small isolated plants and the manner in which they meet the requirements of the equipment which they are intended to supply.

This suitability to the requirements is even more important in these small plants, where the load conditions are practically fixed, than in the much larger public service plants, where the load is much more elastic and may, to a considerable extent at least, be controlled and shaped to meet the capabilities of the power plant equipment by the selling organization of the owners. Many small power plants have been abandoned as unsatisfactory and extravagant, or as luxuries of questionable worth, after a short period of operation, or are being run at excessive cost and with little satisfaction merely because the apparatus is there and no other source of electrical energy is at hand, for no other reason than that the

apparatus installed is not of the type or quality required for satisfactory generation of electrical energy or that the capacities of the various units of the plant equipment are not well suited to each other or to the load conditions to be met.

For this reason it is very inadvisable to decide this question in an offhand manner, or, after having decided upon the installation of a power plant, to leave the details of its equipment to the contractor, for, no matter how honest he may be in his dealings with the purchaser, the planning of such work is out of his line, and he cannot afford to give it the time which it requires, nor has he the facilities for doing engineering work of this nature.

That electric light is much cleaner and more convenient than kerosene lamps is, of course, admitted by all. It is also admitted that a kerosene lamp has too many of the characteristics of a stove to make it pleasant company on a warm summer evening. When it comes to a matter of dollars and cents, there is a general belief that kerosene is considered cheaper, as a source of illumination. This is too often allowed to go uncontradicted, and many times even electric light men will accept this view without taking the trouble to go into a close investigation of the matter.

To thoroughly compare the cost of the two, it is well to start with the assumption that the house in question is to be lighted electrically and calculate the cost of lighting on this basis. We know how much current each lamp will require per hour and the rate at which the current will be purchased, therefore, the rest of this part of the calculation is simple. Next, we must assume that we will install an oil lamp of candle power equal to that of the electric lamp in each place that had been selected for an incandescent lamp. With this same number of kerosene lamps burning the same number of hours will give a figure, for cost of oil plus cost of keep-up in the form of new chimneys, wicks, time of trimming, filling, etc., which will total quite as much as the cost of the electric current, particularly so, as with the oil lamp it is necessary to kalsomine or decorate about every season in order to keep the rooms in a clean condition. This latter cost is necessarily large, and, usually, otherwise unnecessary and should be charged against the kerosene lamp, as a source of illumination.

It is, however, in the application of electricity to labor saving devices, that the greatest gain is to be derived on the farm or country place; and, unless the number of lights to be used each

night averages twenty-five or more throughout the year, it would not pay to install a generating plant for lighting only, without it be confined to the simplest possible equipment. This might consist of an engine and dynamo, but such an equipment could not be conveniently and economically operated to give twenty-four hour service.

There has recently been some activity in the direction of developing the primary battery for commercial use in the especially small isolated places where lighting by electricity is much desired and only a small outlay in first cost will be permitted. The writer has, himself, spent about two years in studying the primary battery situation to determine the feasibility and practicability of utilizing for private isolated plant work, this seemingly convenient and direct method of generating current. The present state of the art, however, provides us only with forms of apparatus very much enveloped in limitations and to such an extent that the electric generating batteries commercially offered for lighting or power service are more suitable for the experimenter than the user who wishes to confine himself to standard articles of equipment.

It is often said, and quite truly, too, that as a class the farmer is more independent and well-to-do than men of any other walk in life. The reason for this is that he is surrounded with natural resources which make him a producer; and, if he is only industrious, his conditions are bound to improve. What he needs, therefore, is merely the sign post to show him the way to obtain the means by which he may increase the returns from his labors, either in greater output or in higher prices.

Each year our farm produce and crops are being more scientifically dealt with and the benefit is reflected in the annual valuation reports.

Apropos of this condition I quote from a report sent out last August by a N. Y. State Farmers' Club, which met and resolved that: "Within a few years there has been a strong drift from city to country; that farm lands are not depreciating in value, but are slowly rising in price; that the farmers receive from one-fourth to one-third more for all produce than they did ten or fifteen years ago, and that there are no abandoned farms in the State."

The agriculturist's position, therefore, is no different from that

of the manufacturer, who has to first supply himself with the machines and tools of the most approved pattern before he is ready to turn out the finished product with which he expects to meet competition.

To infer that the farmer is not financially able to properly equip himself with the most approved and scientific devices for the successful and economical operation of his industry would be as far from the truth as anything we could say.

With lands slowly rising in price the farmer is not offering bad security, if he borrows against his property, in order to equip his farm so that he can produce as much as his neighbor, but at a lower cost, or more than his neighbor, at the same cost and obtain better prices through improved quality of produce.

To give some idea what may be accomplished, I will describe to you the equipment of a private electric generating plant, which one of my clients has at his farm on Long Island, and which serves as a fair example of a plant that would meet the needs of many similar establishments, provided, of course, due allowance be made one way or the other to take care of local conditions.

This farm, which consists in all of about three hundred acres, a portion of which is salt marsh land, not permitting of cultivation, was for years allowed to grow wild, and it was not until about two and a half years ago that the present owner bought the property, and proceeded to develop it along modern lines.

The house was rebuilt, and provided with modern plumbing and heating, and was wired for electric lights throughout. This latter feature, of course, required that connection be made to some source of electric current supply.

The number of incandescent lamps, with which the house is equipped, is about one hundred and sixty, and besides these it was desired by the owner that electricity be provided for light power work in the house and in the farm barn. In the former a motor was installed for operating an organ blower, also one for a small lathe and polishing wheel.

All the machines which the owner had in his barn, with one or two exceptions, were old, but were easily cleaned up, and by supplying a few missing parts, etc., they were arranged for belt drive from a countershaft.

They consisted of a feed grinder, root cutter, fodder cutter, fanning mill, grind stone, circular saw, corn sheller, a small drill

press and a horse clipper. A very nice arrangement of these was made by grouping, so that they can all be driven from one countershaft, which in turn is belt driven by a 3 H. P. motor.

The fact that no two of the larger of these machines are apt to be needed at the same time makes it possible for so many to be taken care of with this one motor.

The barns consist of a number of buildings grouped in "L" shape, the building containing the above mentioned farm machinery being at one end and a small carriage shed at the other. The intermediate buildings are: the creamery, carriage house, horse stable, wagon sheds, etc. The cow barns and poultry houses are separate from the other buildings, and tend to close in the quadrangle.

The house is about six hundred and thirty-five feet from the nearest farm building.

The distance from the center of the group of farm buildings to the entrance to the place, or the shortest distance to the nearest highway, along which the Public Lighting Company's wires are located, is 1900 feet.

The problem, therefore, which had to be solved was: whether the rate at which the Public Lighting Company would supply current, plus the interest on the cost of running a private feeder line this 1900 feet, would make an annual expense greater or less than the yearly cost of operation of a private generating plant of sufficient capacity to take care of the electrical equipment described above, and also a few other useful devices, which at once suggested themselves, to which the electric current is particularly adapted as a source of energy.

A study of the lighting installation planned for this particular residence, with its main stable and farm buildings, indicated a probable average lamp load on ordinary evenings of sixty-four lamps in use for four hours, and on special evenings a maximum load of possibly twice this amount.

To supply the necessary electric current for this lighting load, together with that for the three or four power motors, which the owner had planned for, would have involved a yearly cost for current alone, if taken from local lighting company, of between $1000 and $1100, and besides this, the interest on the cost of installing the private feeders to connect with the Lighting Company's mains, also the cost of maintaining this feeder line.

By installing a private plant, the cost of which in this case amounted to about $3500, the yearly cost of operation figured out approximately $670. This being made up of fuel, lubricating oil, waste and minor repairs, $250, and interest and depreciation at 12%, or $420.

As this plant was so planned and installed that its operation should be practically automatic, no charge is included for attendance. The person looking after the plant need not necessarily be a skilled mechanic, and his duties would not be much more than to each day start up the engine and generator for either the direct service or for charging a storage battery with reserve energy.

The time for starting and the length of the run being predetermined, the attendant is free to attend to other duties most of the time.

It will be seen from the above that the cost of generating the electric current with this private plant is less than two-thirds the amount which the owner would have had to pay the Lighting Company.

Aside from the actual money saved there are sufficient practical advantages to justify a similar decision in favor of the private plant in most country estates when the demands for electric current approach those in this case.

This is particularly true where the demand for power amounts to considerable, for it is very seldom that the service of a public lighting company, to country districts, provides other than alternating current. With the alternating current the methods by which motor speeds are controlled are not so flexible as those for the direct current and, consequently, it is not possible to regulate and to govern the speed at which the various mechanical devices shall be operated. This feature is often quite an important one, as for instance in pumping water, separating cream, grinding feed, etc. With a private plant the generating apparatus can generally be placed in a location which is approximately central with respect to the buildings to which it is to supply current, and the length of the feeders, as a rule, is not so great as to make distribution at low tension prohibitive.

Before passing to some of the uses to which electricity may be easily and profitably applied in the field of the agriculturist, I will describe, briefly, the equipment of the plant, which I have used as an illustration. This may assist some of those present to obtain a

better idea of what such a plant consists, and how satisfactory results may be reached without the outlay of a large sum of money.

The building which was selected for the power plant is that forming the westerly end of the "L" formed by the group of farm buildings. While this location is about 250 ft. from the building containing the farm machinery and the creamery, adjoining, it is the nearest farm building to the residence; being, as mentioned before, about 635 ft. distant.

This location put the site for the power plant about central, with respect to the load to be carried, and it permitted of transforming an unused carriage shed into a power plant at a trifling cost by the substitution of a concrete floor for the old wooden one, and the erection of a three-way partition dividing the original floor space into three.

The building is a one story structure, approximately 20 ft. wide and 30 ft. long, and when partitioned it gave a front room of 15 ft. by 20 ft. to be used as an automobile garage and supply storage, and the partition at right angles to this, gave, in the rear, a room of 10 ft. by 15 ft. for engine and generator, and one of like dimensions for a storage battery.

The only additional work involved in transforming this building from an abandoned shed, such as is to be found in almost any group of farm buildings, was to bring into the building a water pipe connection, make the concrete floor of the engine room four feet deep under the engine and generator and the addition of an air-vent or ventilator over the battery room.

A study of the probable conditions of load, which this plant would have to meet, indicated that the most economical method, both in first cost and operating expense of supplying current for the average load, would be to operate an internal combustion engine generating set during the evening and feed the lights directly from the dynamo. In order, however, to take care of the motor load, which would have to be met at irregular intervals, and to provide a continuous source of supply during the entire twenty-four hours of the day and night, which is of course very desirable, and to provide an auxiliary for the generating set to help carry the load upon special evenings, or the entire load in case of the generating set being shut down for any cause, a storage battery of suitable capacity was installed.

The engine selected was of the internal combustion type, using

crude petroleum, or some of its products readily obtained in the open market in barrel lots. The full load capacity of the engine is ten horse power, and that of the electric generator is five kilowatts at 125 volts, the current of course being direct. These two machines were directly connected, the armature of the generator being mounted on an extension of the shaft of the engine and both mounted on one cast iron base.

The capacity of the storage battery installed in this plant is 288 ampere hours, or 36 amperes for 8 hours, which is sufficient to take care of all the day load, the night load from bed time until morning, and provide a constant reserve source of energy at all times.

As I have stated in the beginning of this paper, many isolated plants have been considered unsuccessful owing to an ill-proportioning of the apparatus. In a large percentage of these failures the trouble has been due to the selection of too small a battery and too large a generating set. The result being that the battery is always in need of being charged and the generating set represents an unnecessarily large first cost, and, as it is too large for practically all load conditions, it has to operate with a low degree of efficiency.

This condition is avoided by installing a storage battery of sufficient capacity, when fully charged, to supply all ordinary demands for current for a period of two to three days, and a dynamo of suitable size for charging the battery at its normal rate.

The combined output of the battery and dynamo must, of course, be sufficient to supply the maximum load that the plant will ever have to carry.

A power plant of this general description requires absolutely no attention other than an occasional brief inspection, except when it becomes necessary to charge the battery or when there is a special demand for current.

The operation of charging the battery requires only occasional attendance upon the plant, and while this must be given regularly, a very large proportion of the attendant's time may be put in at his regular duties while the battery is charging. The location of the power plant frequently has much to do with economy of the attendant's time in this connection.

The engine used in a plant of this description should be of a type especially suited to the driving of electrical machinery. It

is highly important that its speed should show very little variation under conditions of changing load and throughout the cycle of operation of the engine. In no other way can satisfactory electrical service be obtained.

The engine should also, it is needless to say, be economical in fuel consumption and in the use of lubricating oil and other necessary supplies, and should be as nearly absolutely reliable in its operation as is possible.

The dynamo for such a plant may be either of the ordinary compound wound type, or it may be of the variable voltage type, with compound field winding, capable of operating directly upon the line as a compound wound machine, or of operating as a shunt wound machine when used for charging the battery, delivering current to the battery, at any required voltage up to forty per cent. above normal. The former is slightly simpler, as far as the dynamo is concerned, and gives better regulation of voltage, but it requires that a small motor dynamo or booster set be provided for raising the voltage for battery charging; whereas, the latter method eliminates the expense of a booster without going to the other alternative of dividing the battery and charging through a resistance, which is an inefficient method of operation, and requires either a larger dynamo and engine, or a doubling up of the time necessary for charging.

A very vital point to be considered in connection with any plant operating a storage battery is the regulation of the voltage at which the current supplied by the battery on discharge is delivered to the line. Some means must be provided by which this voltage will be kept constant or cause to vary slightly in proportion to the load carried, under all conditions of loading and wholly independent of the decrease in the voltage of the battery itself as its charge is converted into electrical energy and withdrawn.

There are a number of ways in which this may be done, but, for a plant of this description, where a minimum of attention is essential, and yet where the voltage regulation must be as nearly perfect as it is possible to have it, the most satisfactory method of regulating the voltage is by means of a variable resistance placed in series with the battery and automatically operated through the medium of a relay connected across the line.

This relay may be differentially wound so as to produce an

over-compounding effect and in this way may overcome the loss in pressure between the switchboard and the lamps, and also allow of the operation of the battery and dynamo in multiple, in the same manner that two compound wound generators may be operated.

Such a voltage regulator can be obtained at a cost entirely commensurate with that of other methods of controlling the voltage, and the results obtained, in a plant of this nature, are far more satisfactory.

On farms and country places of considerable size, it is usual to have what is ordinarily known as a handy man, who is more or less familiar with the machinery, its care and operation. Such a man is almost indispensable for the care and operation of automobiles, launches, labor-saving machinery, the heating apparatus in the residence, greenhouses and other buildings, and the general up-keep of the various mechanical and electrical devices.

Such a man may readily, and to great advantage, be placed in charge of the care and operation of the power plant, as this will require so little attention as not materially to increase his duties.

Water power that can be relied upon for sufficient energy, of course, makes the least expensive source of power; but, if a suitable water power is not available, an engine of some kind must be provided. A consideration of the two general classes, namely: those driven by steam and those in which the fuel is burned within the engine cylinder will usually result in a choice of one of the latter type, as with the former the necessity for a high pressure boiler, with its smoke stack, adds considerably to the cost of the equipment, and the power house building and the greater complication of the steam plant requires a greater expenditure for labor than is necessary for a plant in which the prime mover is an internal combustion engine.

An exception to this might be, of course, in large dairy farms, or other places where it is of importance that there shall always be plenty of real hot water on hand for cleaning purposes.

In deciding upon the particular kind of internal combustion engine to install, that is, whether it shall operate on producer gas, natural gas, or illuminating gas, or gasolene, kerosene, or crude petroleum, a conclusion should be reached only after a careful consideration of duration of the load, the relative costs of the

various fuels at the point where the plant will be located, and the possible fire hazard of one or another, as well as the relative cost of the various types of engines with their auxiliary apparatus.

The possibility of a disproportionate increase in the cost of any of the fuels considered, or of its becoming difficult or impossible to obtain some one or more of these fuels in the future, should also be carefully considered.

After a power plant has once been installed there are many ways in which it may be used to great advantage, that are perhaps not so fully appreciated by the prospective purchaser, as they are

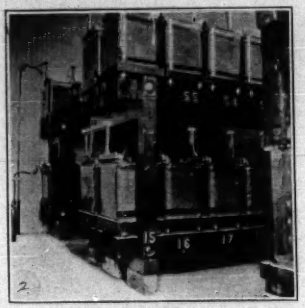


Fig. 1. A direct-connected dynamo and a kerosene oil engine. This generating unit is the one described earlier in this paper in reference to the private plant on Long Island.

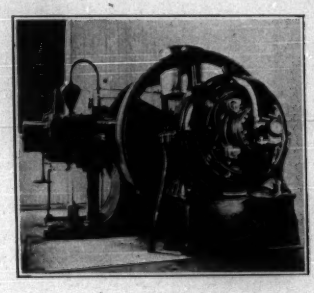


Fig. 2. Electric storage battery, same plant.

by the owner of one of these little plants. It is a fact that the largest items of operating cost in a small plant, designed principally to take care of the evening lighting, are those of interest upon the original investment, and depreciation in value of the equipment as it ages, the cost of fuel and attendance being, comparatively, small items. These facts make it apparent that such current as is used at other times than during the evening, and, therefore, without necessitating any additional investment or greater depreciation charge, may be generated at very small cost.

It is, therefore, possible to add greatly to the conveniences upon a country place equipped with an electric plant, by the installation of electrically operated devices of one kind and another, which would be out of the question if current for their operation had to

be purchased at the prevailing rates for public electrical supply.

Turning to the slides, which I have prepared to illustrate some of the points mentioned this evening, we will see:

This equipment, at the time I last tested it, separated the cream of sixty quarts of milk in thirteen minutes, which makes it possible to skim the milk of five to seven cows, and give the skimmed milk to the calves before it has lost its natural heat.

An electrically driven churn, which was installed in this same creamery, I could not get a photograph of, but its duty was to turn

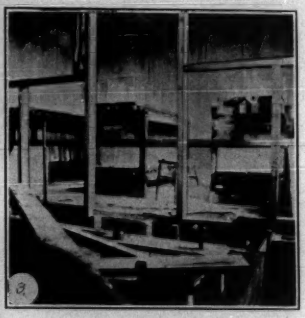


Fig. 3. Same battery, partly assembled, foundation for generating set in foreground. Partition separating battery room from engine room only partly completed.

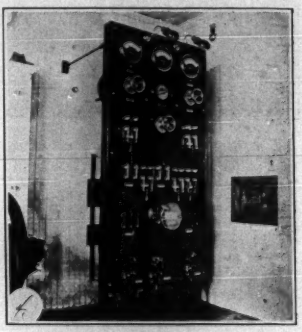


Fig. 4. Switchboard, showing station voltmeter in center at top of board with the battery ammeter on one side and the dynamo ammeter on the other. A recording watt hour meter is further down on the board, just above the circuit breakers.

out, daily, ten to twelve pounds of butter, which it would do in fifteen minutes.

An equipment, which can often times be installed and which will afford much convenience and profit, is that of electrical refrigeration. This is an important item and very little accurate information has ever been published on this subject. One reason is, that with the ordinary refrigerating machine the necessity of more or less constant attention has made the equipment so much of a nuisance that in many instances the operating expense made the use of the small refrigerating machine prohibitive.

To be practical for use on either the farm or the country estate we must have a refrigerating machine which does not involve a

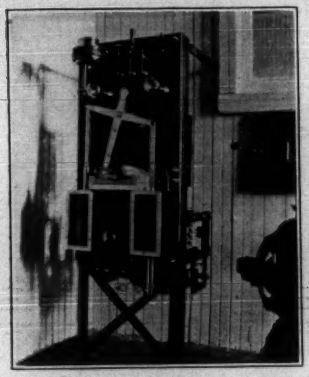


Fig. 5. While this view shows the voltage regulator installed in the Long Island plant, it is in its principal characteristics the same as I would use in any such plant. The panel itself is constructed like a switchboard. The differentially wound relay is seen at the top of the board. Its potential coil is wound for the station bus bar voltage. The differential coil and its shunt are so wound and adjusted as to counteract the natural drop of voltage of the system, which would otherwise occur, as the load passes from zero to the full load.

This device will control the voltage of the current supplied to the switchboard by the battery within one-half volt, above or below, that for which the regulator is set, and it will do this under any condition of battery voltage.

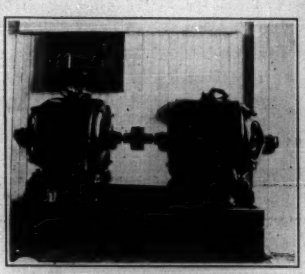


Fig. 6. The booster set consists of a direct current shunt wound motor coupled to a direct current separately excited generator.

Fig. 7. Rear view of the power plant building.

Fig. 8.

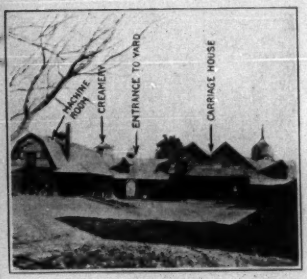


Fig 9.

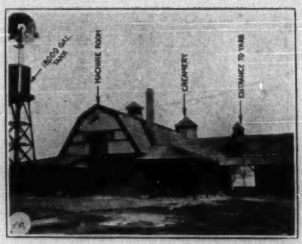


Fig. 10.

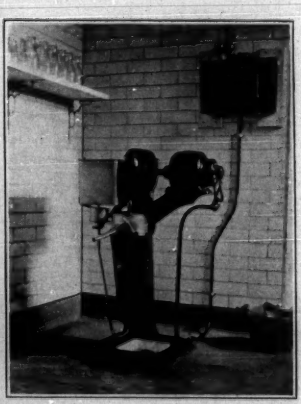


Fig. 11. Interior view in creamery, showing a Tubular Cream Separator driven directly by an electric motor. This equipment, as you will see, is well put up. It consists of a Crocker-Wheeler motor mounted on a bracket attached to a Sharples Separator.

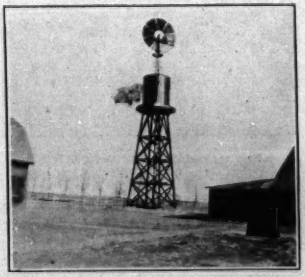


Fig. 12. This view shows the one great mistake of this otherwise ideal farm installation. This 18,000 gallon tank cost about as many hundred dollars, and could have been replaced with a one-horse power electric pump equipment, which would not have cost over two hundred dollars, and would have been much simpler, more reliable, and better suited to the owner's needs in every way.

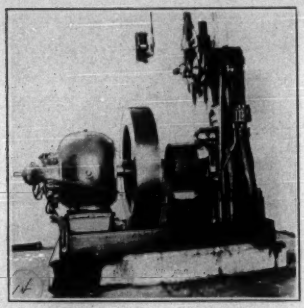


Fig. 14. Generating set of another plant.

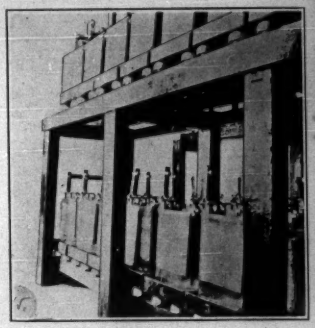


Fig. 15. Battery room for same.

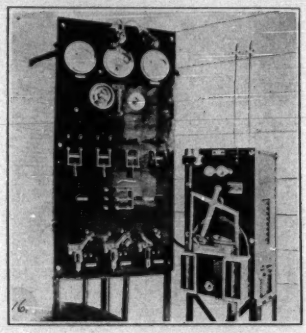


Fig. 16. Switchboard and voltage regulator.

Fig. 17. Power plant building, inexpensive type. Combined with it are garage and living quarters for men.

Fig. 18. Residence containing the equivalent of about one hundred 16c.p. lamps.

high first cost or a high cost of maintenance. one which does not require the attention of a skilled mechanic and one of sufficiently compact proportions to permit of being housed in an inexpensive structure or out-building.

The automatic refrigerating machine fills these requirements. Equipments of this type are based on the direct expansion system.

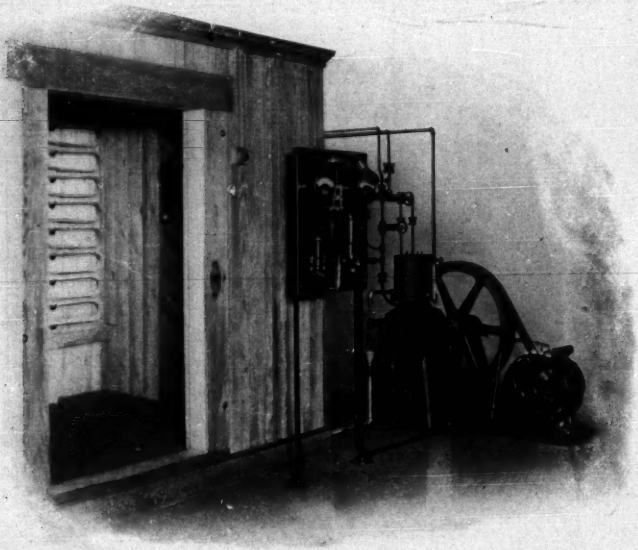


Fig. 19, illustrates the electrically operated automatic machine, refrigeration chamber and expansion pipes.

In carrying out the cycle of operation of the ordinary non-automatic refrigerating machines the compressors have to be oiled; the flow of water to the condenser has to be watched day and night to avoid the occurrence of dangerous pressures; the starting and stopping of the machine and the regulating of the flow of liquid ammonia through the expansion valves, has to be done by a skilled attendant.

Cold storage by means of ice has been made the subject of study

by many inventors and engineers, but this method in all its forms carries with it three inherent defects: the presence of considerable moisture or dampness, the impossibility of regulating and controlling the temperature and the constant waste of energy.

In isolated places, where no electric current is either generated or to be had from the public supply service, some form of gravity brine system would be more economical to adopt for the refrigera-

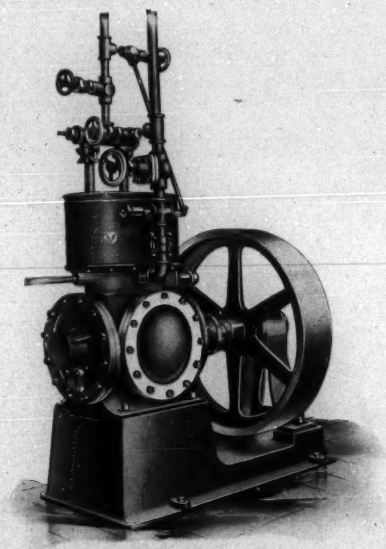


Fig. 20 shows the ammonia compressor alone.

tion of ordinary farm produce, such as eggs, butter, milk, cream, etc., than to attempt to run a refrigerating machine by power.

Where the electric equipment, however, has already been adopted for other purposes it is a very simple matter to extend its use so as to take care of the requirements of operation of an automatic refrigerating machine.

As the name implies, these machines take care of themselves, and they will maintain a temperature which shall not vary more

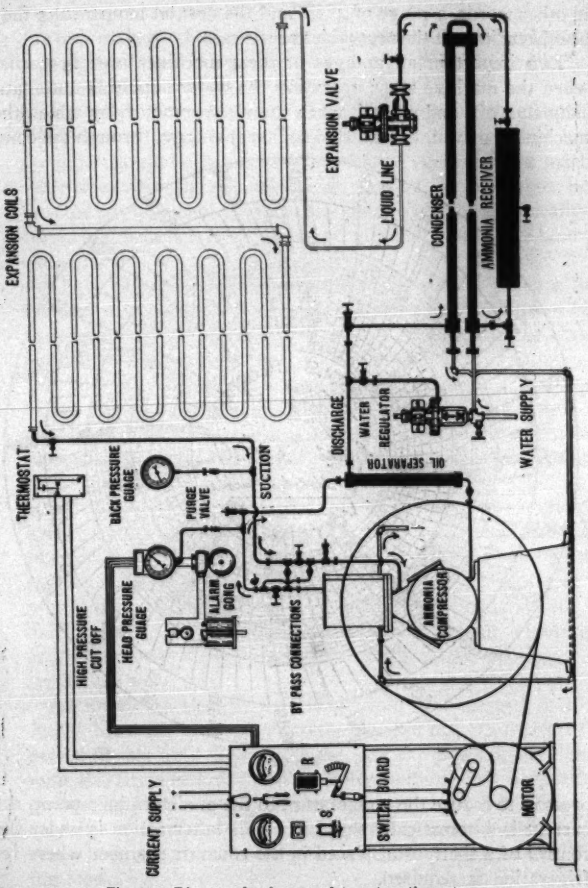


Fig. 21. Diagram showing complete automatic system.

than two degrees either up or down from the desired point, or, in other words, a range of 4°F., and the desired temperature may be selected to suit the necessities of the case.

Two important advantages of these machines are: first, that when the machine stops operating the water supply is automatically shut off, and second, when the electric motor, by which the machine is driven, has exerted sufficient energy, through the com-

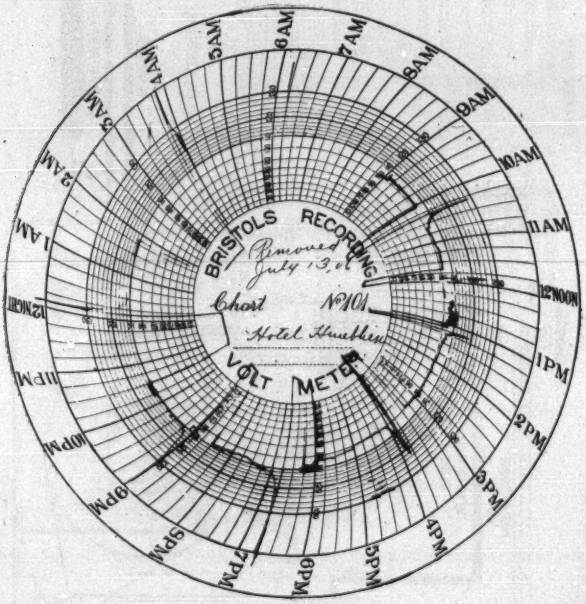


Fig. 22.

pressor, to reduce the temperature to the predetermined point, the current is automatically shut off. This latter action is under the control of a thermostat placed in the room or chamber where refrigeration is required.

A very pretty illustration of the saving which these machines give by automatically cutting off not only the supply of water but also the current during the periods when no energy is neces-

sary to maintain the desired degree of cold, is shown on slide, Fig. 22.

These records show that the compressor was in operation for only 75% of the total twenty-four hour period, during which the test was made.

Generally, a non-automatic machine must be shut down at night and the process of refrigeration continued by brine, but as the inherent efficiency of the brine system necessitates a machine more than twice the normal size on account of the shorter hours the cost of equipment is practically doubled. The electrically operated automatic machine, however, gives a twenty-four hour run unattended.

I have given the refrigerating machine subject perhaps more than its share of time this evening, but I think it will not be in vain, as it is, in my opinion, a most important one to the class of agriculturists for whom this paper is intended.

A point which I believe I have stated before, and which I wish to lay stress on, is that the successful farmer of the future can only be the man who will face his problems squarely and provide such means as may be necessary to turn out either better quality or the same quality at a lower cost.

Let us assume, therefore, that for other reasons an electric plant has been installed, it then simply becomes a matter of calculation to determine whether or not by having the means at hand whereby produce may be kept in proper condition from a period of low prices to one of high values, the labor saved in harvesting and daily handling ice, and the more healthy condition of using ice artificially made from pure well or spring water, justify the outlay for the apparatus.

As far as the electric current required is concerned, it will be found, by a reference to the weather bureau reports, that the daily period of day light throughout the year corresponds very closely with the temperature. Consequently, with a well proportioned generating plant it would not be necessary to provide any greater capacity to take care of the refrigerating machine than had already been provided for the other power apparatus and the lighting load.

Almost any dry, clean cellar will serve as a cold storage room.

Apples can be kept in this way in perfect condition until the price has gone up high enough to net probably twice as much as

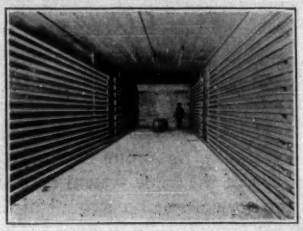


Fig. 24. Refrigeration coils on cellar walls.

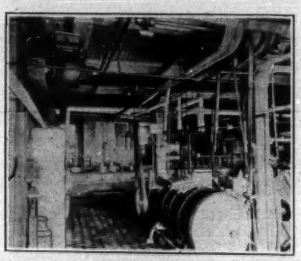


Fig. 30. "Northern" motor driving churns, milk heater, butter worker, etc., in modern dairy.

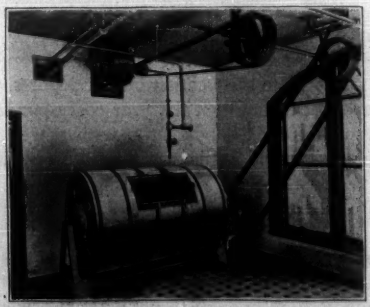


Fig. 31. "Northern" motor driving Fargo churn.

it would be possible to obtain at the time of picking the fruit. This, of course, applies equally well to all fruits, poultry, eggs, butter, etc.

The many little domestic electrical devices, such as flat irons, plate warmers, sauce-pan heaters, sewing machines, fans, etc.,

Fig. 32. "Northern" motor driving bottle washing machine and pasteurizing apparatus in a modern dairy. Pasteurizing equipment in the room at the right.

Fig. 33. "Northern" backgear motor driving grindstone and deep well pump.

which have from time to time found their way to the public, are too well known for me to more than mention, and in the stable; electric horse clippers, dental floats and grooming machines are to be seen in daily operation. But electrically operated pneumatic floor sweepers, ice cream freezers, dough mixers, churns, cream

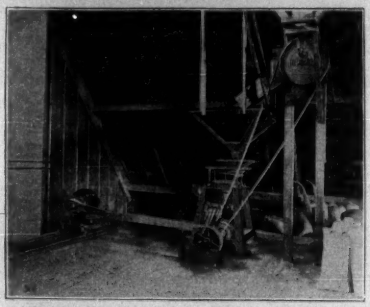


Fig. 34. "Northern" motor driving Nordyke & Marmon corn mill.

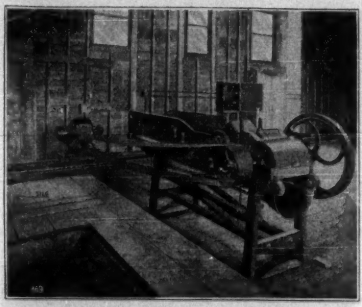


Fig. 35. "Northern" motor driving Smalley ensilage cutter in dairy barn.

Fig. 36. "Northern" motor mounted on truck in dairy barn. This motor serves to drive both the corn mill in Fig. 34 and the ensilage cutter in Fig. 35. It travels on rails, making it easy to set in running position and easily kept in alignment.

separators, feed mixers and grinders of all kinds, milking machines and chicken hatchers, are things we do not see every day.

I have been fortunate enough to obtain good photographs of many of these, and I have had slides made to show you that they are not mere fancy, but are practical and their use will, before a great while, be very general.

One can hardly glance at the newspapers nowadays without seeing an account of some serious conditions which have been brought to light through our modern day methods of testing cows and their milk for the presence of disease germs.

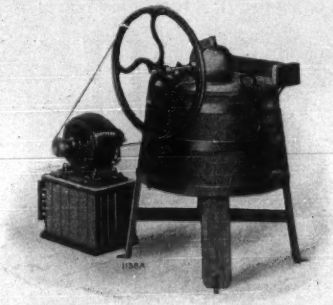


Fig. 37. "Northern" motor driving "Ocean-wave" washing machine.

I do not think that the fact that we hear a great deal about this sort of thing indicates that the average milk supply is to-day worse than it used to be, but people are getting to realize more the necessity for strictly sanitary conditions, and are demanding improvements which will insure that they may have them.

Sanitary milk, therefore, is worth considerably more than that in regard to which there is question.

The thing which has always been a problem to me is that under the conditions which obtain in, I may say, a majority of the dairy barns, that there are not many times more disease germs in the milk than bacteroligists report.

The milking machine has opened up a new idea in this direction and, in my opinion, represents a most decided step in advance in farm practice. In fact, I do not hesitate to prophesy that it will, before a great while, revolutionize the milking business.

The principles upon which the best mechanical milking systems have been developed may be described briefly as follows: The cow, with all its peculiarities in regard to giving up her milk, is, through the agency of mechanical devices, connected directly through a system of tubing, with the milk bottle itself, if you please, and is made to give up her milk, which she does, if the machine is properly applied, more willingly and in greater quantity than she would to the average hand milker.

It will be seen, therefore, that there is no possible way in which germs can enter the milk from outside sources from the time the milk leaves the cow until it is bottled or put up in shipping cans.

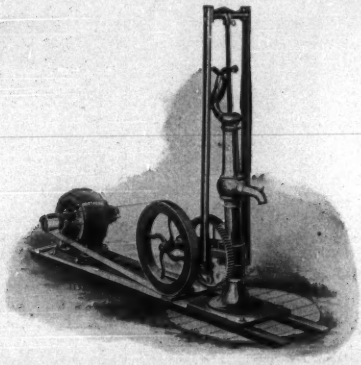


Fig. 38. "Northern" motor driving Fuller & Johnson No. 3 pumping jack.

This statement is, of course, made on the assumption that the mechanical devices and conveyor tubing shall at all time, when not in use, be kept sterilized, which, of course, they should be.

Some skeptics may say this cannot be done, but my answer is that, if a surgeon cannot keep his instruments clean, he is not qualified to practise surgery.

It consists essentially of a strong tin pail, weighing about twelve pounds and holding 54 pounds of milk, smaller at the top than at the bottom, with cast metal cover fitting loosely, but resting on a rubber gasket, which makes an air-tight joint. On the cover is mounted the pulsating mechanism, and from it project three nipples, on one of which is slipped the hose which connects with

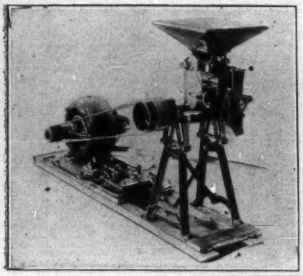


Fig. 39. "Northern" motor driving No. 6 duplex corn mill.

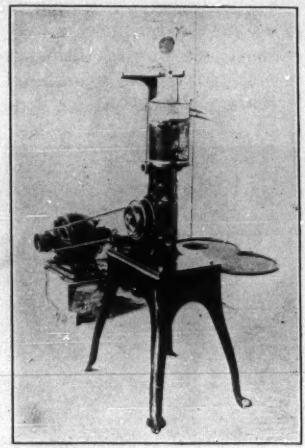
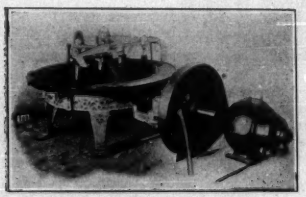


Fig. 40. "Northern" motor driving a DeLaval cream separator.



No. 4.1 "Northern" motor driving Dayton cream beater.

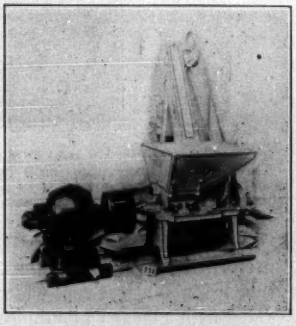


Fig. 42. "Northern" motor driving Marseilles corn mill through flexible coupling.

the stanchion cock in the piping system, and on the other two the rubber tubing which leads to the two cows. These two nipples are provided with cocks. As the milk enters it passes through a short piece of glass tubing, and by watching this we know when milking is completed. The other end of each of these rubber milk-tubes, which are 3½ feet long, is attached to a small metal con-

Fig. 43 shows the milking machine itself.

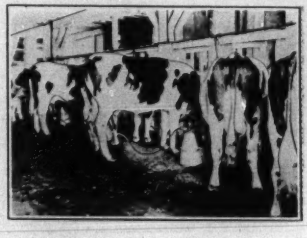


Fig. 44 shows that one machine will milk two cows at a time, and one man can attend three or four machines.

In Fig. 45 you will see the usual litter and feed carrier rail over-head. These carriers are, so far as I know, universally propelled by hand. A plan which I would like to see employed for this would be an electrically driven and controlled telpher. I believe it would tend greatly to decrease the time which is required for feeding and properly cleaning out the cow barn.

nector, which has four outlets, from each of which a short rubber tube leads to a teat-cup, which is made of tinned metal and provided with a rubber mouthpiece to fit around the base of the teat.

There are five sizes of these teat-cups to fit different sized teats, and the chief skill of the operator is in choosing the right size. When these are slipped on over the teats they stay there by at-

mospheric pressure until removed by the attendant. It will be easily understood that the effect of the vacuum is to draw the milk from the teat to the pail; but this is not enough. When this is done the teat is empty, more milk cannot get into it from the udder, and the performance stops. To prevent this a contrivance has been added which breaks the vacuum, and then a fresh supply of milk enters the teat; then the vacuum is created again and this

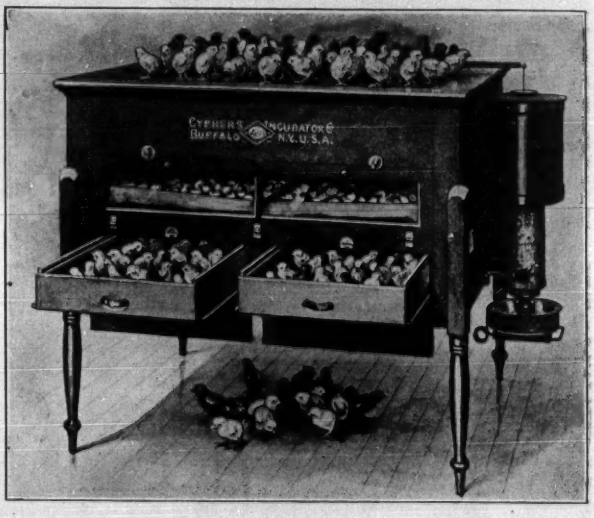


Fig. 46.

additional milk goes to the pail, and so on until the milking is completed.

The vacuum pump must of course be driven by some source of power. The power may be of any kind, electric motor, gasolene engine, tread power or a steam engine. If the steam engine be used, it follows that there will be plenty of real hot water on hand for cleaning purposes, but this advantage, however, may be obtained by having a sterilizer of suitable proportions to perform the work of cleansing. This sterilizer can be heated by coal, gas or electricity.

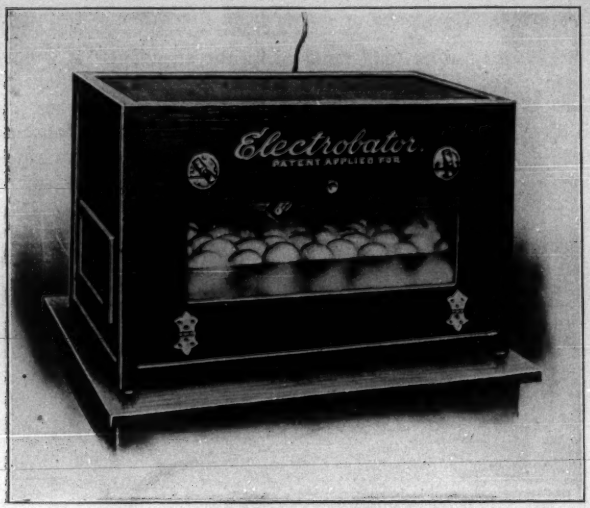


Fig. 47. An electrically heated and regulated incubator.

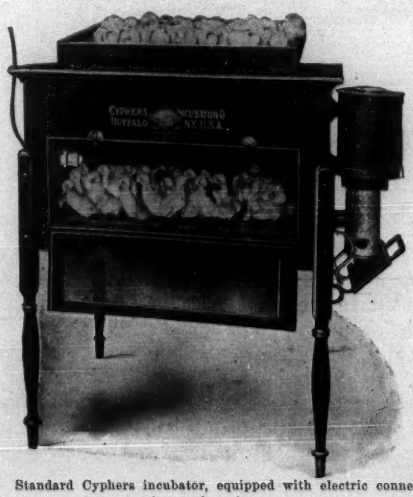


Fig. 48. Standard Cyphers incubator, equipped with electric connection and electro-plane heater

Fig. 49. An electric, self-regulating, fire-proof heater designed for use in Standard Cyphers, No. 0 and No. 1, Incubator, and in other makes of incubators of similar capacity.

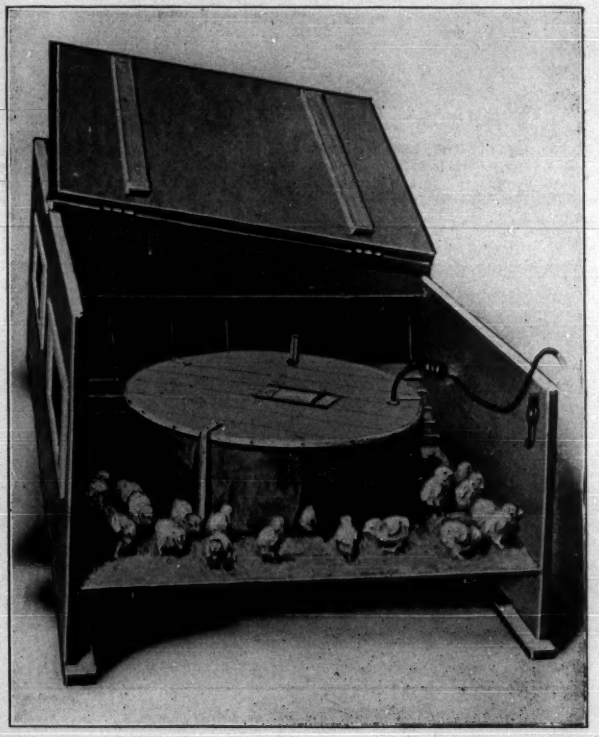


Fig. 50. Electric hover in use in exercising compartment of a Cyphers Colony Brooder.

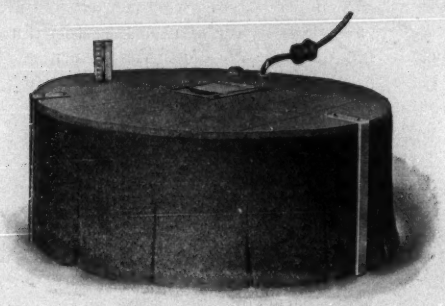


Fig. 51. An electrically-heated, self-regulating brooding device for use in any make of brooder, either in-door or out-door, that is sufficiently large to accommodate a hover twenty-four inches in diameter and ten inches in height.

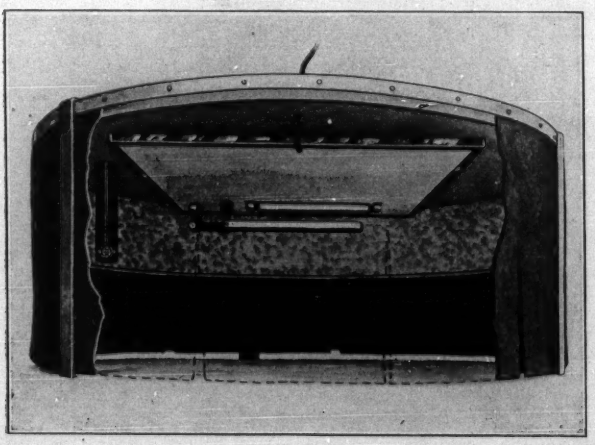


Fig. 52.

The electric motor will without doubt be found to be the most convenient and efficient source of power, as cows are only milked twice a day, and those in attendance have other duties to perform during the remainder of the day. With the motor the power is on or off when you throw the switch, which is not the case with a steam engine and boiler.

The cow-milker which my slides illustrate is the "Burrell-Lawrence-Kennedy."

I have not spoken of the economy of these machines, but at a glance it will be seen that the sanitary advantage, while most important, from the public's point of view, is not the only one which will appeal to the dairyman.

Before bringing this paper to a close I must not forget to pay tribute to the ever cackling hen, which no longer is to be allowed to hatch her brood of chicks in peace and comfort. The necessity of increased output has reached her also.

As a result of a series of experiments one of the leading incubator companies has recently placed on the market some ingenious and practical apparatus whereby hatching and brooding of chicks and ducklings is accomplished by electricity obtained from ordinary lighting or power fixtures in residences or other building. The idea is a very simple one, and it has been cleverly worked out, as you will see from the accompanying slides.

TEN THOUSAND DOLLARS AWARDED ART STUDENTS.

With the closing of the year of the Schools of the Pennsylvania Academy of Fine Arts, the oldest and most widely known art school in America, the announcement was made on Thursday, May 28th, of the award to eighteen of its students of travelling scholarships of $500.00 each for four months' travel and study in Europe.

These Scholarships, known as the William Emlen Cresson Memorial Scholarships, are each year available by the Academy, which has already sent forty-three pupils abroad under the Cresson foundation.

The effect upon the art of this country of the policy adopted by the Pennsylvania Academy is bound to be far-reaching, and indeed, is already noticeable in our current exhibitions wherever held.

The scholarships this year were awarded: Two in the Department of Architecture, two in the Department of Illustration, two in the Department of Sculpture, and ten in the Department of Painting.

In addition to the Travelling Scholarships the Academy's prize list is large in all its departments. The principal prizes being the Edmund Stew-

ardson in Sculpture of $100.00, awarded this year to Beatrice Fenton, of Philadelphia; the Henry J. Thouron in competition of $50.00 each, awarded to Nathaniel J. Pousette and Lawrence B. Saint; Henry J. Thouron Prize of $25.00 each, awarded to Henry L. Wolfe and Juan N. Arellano, who comes to the Academy from the Philippine Islands; Charles Toppan Prize of $400.00, to H. Willard Ortlip, of Norristown, Pennsylvania, and the Charles Toppan Prize of $300.00 to Adolph W. Blondheim, of Philadelphia.

The Faculty of the Pennsylvania Academy of the Fine Arts is composed of the following distinguished artists: Herbert M. Howe, M.D., Chairman; Thomas P. Anshutz, Hugh H. Breckinridge, William M. Chase, Henry McCarter, Paul Phillippe Cret, George McClellan, M.D., Cecilia Beaux, Frank Miles Day, Henry R. Poore, W. Sergeant Kendall.

Book Notices.

Electro-Analysis, by Edgar F. Smith, Professor of Chemistry, University of Pennsylvania. Fourth edition, revised and enlarged. 336 pages, illustrations, 12-mo. Philadelphia, P. Blakiston's Son & Co., 1907. Price, in limp leather, $2.50.

The first edition of this work was issued in 1890 as a text-book for students in electro-chemistry. From a book of about one hundred pages it has grown to an imposing volume of three hundred and thirty-six pages.

The present edition has been completely revised, and much new material has been added. Innumerable references to existing literature add to the value of the book. R.

The Blast Furnace and the Manufacture of Pig Iron. An elementary treatise for the use of the metallurgical student and the furnaceman, by Robert Forsythe. 368 pages, illustrations, 8-vo. New York, David Williams Co., 1908. Price, $3.00.

This volume is designed chiefly for the use of beginners. It contains a simple and concise statement of the general principles of the manufacture of iron and the operation of the blast furnace.

Marginal references throughout the book make it possible for the student to inform himself more fully on any branch of the subject.

The author, a graduate of Harvard University, and for three years instructor in metallurgy in the same institution, died while the work was being prepared for the press. It has been carried to completion by a friend, who has followed the plans as outlined. A. R.